All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Design a Turing machine the copies a string. The machine starts with a string $x \in \{0, 1\}^*$ on the tape with the head on the first symbol of $x$. When the Turing machine halts the string $xcx$ is written on the tape with the head on the first symbol of the output. You may use a state diagram as your design, but explain what the various states mean.

2. (10 points) Given a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ that halts on every input, design a Turing machine $M'$ that also halts on every input and $L(M') = \Sigma^* - L(M)$. Recall that final states $q$ have the property that $\delta(q, a)$ is undefined for all $a \in \Gamma$. Thus, it does not suffice to make all the non-final states final.

3. (10 points) Let $G = (V, \Sigma, P, S)$ be a context-free grammar. Using a closure algorithm we know how to compute the productive non-terminals, those non-terminals that generate a terminal string. Assume we have an algorithm for computing the positive productive non-terminals, that is, all non-terminals $A$ such that $A \Rightarrow^* x$ for some $x \in \Sigma^*$. For $A \in V$, define the relation $R(A, B)$ if $A \Rightarrow^* xBy$ for some $xy \in \Sigma^*$ and $R^+(A, B)$ if $A \Rightarrow^* xBy$ for some $xy \in \Sigma^*$.

   (a) Design a closure algorithm to compute $R(A, B)$ for all $A, B \in V$. You may use the algorithm for computing $\text{Prod}$, the productive non-terminals of $G$ as a subroutine.

   (b) Design a closure algorithm to compute $R^+(A, B)$ for all $A, B \in V$. You may use the algorithm for computing the relation $R$, the algorithm for computing $\text{Prod}$, and the algorithm for computing $\text{Prod}^+$ (the set of positive productive non-terminals) as subroutines in your algorithm.

   (c) Using $R$, $R^+$, and $\text{Prod}$ as subroutines design an algorithm for determining if the language generated by $G$ is infinite. Note that $L(G)$ is infinite if and only if there are terminal strings $u, v, x, y, z$ and a non-terminal $A$ such that $S \Rightarrow^* uAz$, $A \Rightarrow^* vAy$ with $vy \neq \epsilon$, and $A \Rightarrow^* x$. 

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