All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Design deterministic finite automata using a state transition diagram for each of the following languages.
   
   (a) \{ x \in \{0, 1\}^* : 010 is a substring of x \}.
   
   (b) \{ x \in \{0, 1\}^* : 111 is not a substring of x \}.
   
   (c) \{ x \in \{0, 1\}^* : x contains exactly 5 0’s \}.
   
   (d) \{ x \in \{0, 1\}^* : x has an even number of 0’s or an odd number of 1’s \}.

2. (10 points) Consider the languages
   
   \[ L_k = \{ x \in \{0, 1\}^* : x \text{ contains exactly } k \text{ 0’s} \} \]
   
   for \( k \geq 0 \).
   
   (a) Formally define a deterministic finite automaton \( M_k \) with exactly \( k + 2 \) states that accepts \( L_k \).
   
   (b) Prove by contradiction that every deterministic finite automaton that accepts \( L_k \) has at least \( k + 2 \) states. The ideas from problem 2 of assignment 1 are useful.

3. (10 points) A finite state transducer \( M = (Q, \Sigma, \Gamma, \delta, q_0) \) is defined by: \( Q \) is a finite set of states, \( \Sigma \) and \( \Gamma \) are alphabets and \( \delta : Q \times \Sigma \rightarrow Q \times \Gamma^* \). That is, \( \delta(q, \sigma) = (p, y) \) means that on input \( q \) processing \( \sigma \in \Sigma \), \( M \) goes to state \( p \) and outputs the string \( y \). Let \( w = w_1 w_2 \cdots w_n \) where \( w_i \in \Sigma \). We write \( p \xrightarrow{w,y} q \) if there are states \( r_0, \ldots, r_n \) and \( y_1, \ldots, y_n \in \Gamma^* \) such that:

   \[
   \begin{align*}
   y &= y_1 y_2 \cdots y_n, \\
   r_0 &= q, \\
   r_n &= p, \\
   (r_i, y_i) &= \delta(r_{i-1}, w_i), \ 1 \leq i \leq n.
   \end{align*}
   \]

   For \( x \in \Sigma^* \), define \( f_M(x) = y \) if \( q_0 \xrightarrow{x,y} p \) for some \( p \in Q \). The string \( f_M(x) \) is called the output of \( M \) on input \( x \).
Design a finite state transducer that outputs the quotient in binary of a number written in binary divided by 3. For example, the quotient of 11 divided by 3 is 01 because 11 is 3 written in binary. Another example is the quotient of 1101 divided by 3 is 0100 because 1101 is 13 written binary and 0100 is 4 written in binary.