Simulating Nondeterministic TMs

- **Nondeterministic TMs (NTMs)**
  - $$\delta : Q \times \Gamma \rightarrow \text{Pow}(Q \times \Gamma \times \{L,R\})$$
  - No $$\varepsilon$$ transitions but can simulate them by reading and writing same symbol and moving head back to same position
- Any nondeterministic TM N can be simulated by a deterministic TM M
- N accepts w iff there is at least 1 path in N’s tree for w ending in $$q_{\text{ACC}}$$
  - Proof idea: Use breadth-first search to simulate each branch
    - Explore all branches at depth $$n$$ before $$n+1$$

Simulating Nondeterminism: Details, Details

- Use a 3-tape DTM M for breadth-first traversal of N’s tree on w:
  - Tape 1 keeps the input string w
  - Tape 2 stores N’s tape during simulation along 1 path (given by tape 3) up to a particular depth, starting with w
  - Tape 3 stores current path number
    - E.g. $$\varepsilon$$ = root node $$q_0$$
    - 213 = path made up of 3rd child of 1st child of 2nd child of root
- See text for more details
Closure Properties of Decidable Languages

- Decidable languages are closed under $\cup$, $\circ$, $\ast$, $\cap$, and complement
- Example: Closure under $\cup$
- Need to show that union of 2 decidable L’s is also decidable
  Let M1 be a decider for L1 and M2 a decider for L2
  A decider M for $L_1 \cup L_2$:
    On input w:
      1. Simulate M1 on w. If M1 accepts, then ACCEPT w. Otherwise, go to step 2 (because M1 has halted and rejected w)
      2. Simulate M2 on w. If M2 accepts, ACCEPT w else REJECT w.
  M accepts w iff M1 accepts w OR M2 accepts w
  i.e. $L(M) = L_1 \cup L_2$

Closure Properties

- Consider the proof for closure under $\cup$
  A decider M for $L_1 \cup L_2$:
    On input w:
      1. Simulate M1 on w. If M1 accepts, then ACCEPT w. Otherwise, go to step 2 (because M1 has halted and rejected w)
      2. Simulate M2 on w. If M2 accepts, ACCEPT w else REJECT w.
  M accepts w iff M1 accepts w OR M2 accepts w
  i.e. $L(M) = L_1 \cup L_2$

Will this proof work for showing Turing-recognizable languages are closed under $\cup$? Why/Why not?
Closure for Recognizable Languages

- Turing-Recognizable languages are closed under $\cup$, $\circ$, $\ast$, and $\cap$ (but not complement! We will see this later in Chapter 4)
- Example: Closure under $\cap$
  Let $M_1$ be a TM for $L_1$ and $M_2$ a TM for $L_2$ (both may loop)
  A TM $M$ for $L_1 \cap L_2$:
  On input $w$:
  1. Simulate $M_1$ on $w$. If $M_1$ halts and accepts $w$, go to step 2. If $M_1$ halts and rejects $w$, then REJECT $w$. (If $M_1$ loops, then $M$ will also loop and thus reject $w$)
  2. Simulate $M_2$ on $w$. If $M_2$ halts and accepts, ACCEPT $w$. If $M_2$ halts and rejects, then REJECT $w$. (If $M_2$ loops, then $M$ will also loop and thus reject $w$)
  $M$ accepts $w$ iff $M_1$ accepts $w$ AND $M_2$ accepts $w$ i.e. $L(M) = L_1 \cap L_2$