The Pumping Lemma for Regular Languages

♦ What is the idea behind it?
   ♦ Any regular language $L$ has a DFA $M$ that recognizes it
   ♦ If $M$ has $p$ states and accepts a string of length $\geq p$, the sequence of states $M$ goes through must contain a cycle (repetition of a state) due to the pigeonhole principle! Thus:
   ♦ All strings that make $M$ go through this cycle 0 or any number of times are also accepted by $M$ and should be in $L$.

Formal Statement of the Pumping Lemma

♦ Pumping Lemma: If $L$ is a regular language, then there exists a number $p$ (the “pumping length”) such that for all strings $s$ in $L$ such that $|s| \geq p$, there exist $x$, $y$, and $z$ such that $s = xyz$ and:
   1. $xyz \in L$ for all $i \geq 0$, and
   2. $|y| \geq 1$, and
   3. $|xy| \leq p$.

♦ On board proof…(see page 79 in textbook)
Pumping Lemma in Plain English

- $p =$ number of states of a DFA accepting $L$.
- Any string $s$ in $L$ of length $\geq p$ can be expressed as $s = xyz$ where $y$ is not null (y is the cycle), $|xy| \leq p$ (cycle occurs within $p$ state transitions), and any “pumped” string $xy^iz$ is in $L$ for all $i \geq 0$ (go through the cycle 0 or more times).

Using The Pumping Lemma

- **In-Class Examples:** Using the pumping lemma to show a language $L$ is not regular
  - 5 steps for a proof by contradiction:
    1. Assume $L$ is regular.
    2. Let $p$ be the pumping length given by the pumping lemma.
    3. Choose cleverly an $s$ in $L$ of length at least $p$, such that
    4. For any way of decomposing $s$ into $xyz$, where $|xy| \leq p$ and $y$ isn't null,
    5. You can find an $i \geq 0$ such that $xy^iz$ is not in $L$.

- **Example 1:** $\{0^n1^n \mid n \geq 0\}$
Proving non-regularity as a Two-Person game

- An alternate view of using the pumping lemma to show a language $L$ is not regular
  - Think of it as a game between you and an opponent (KB):
  1. **You**: Assume $L$ is regular
  2. **KB**: Chooses some value $p$
  3. **You**: Choose cleverly an $s$ in $L$ of length $\geq p$
  4. **KB**: Breaks $s$ down into some $xyz$, where $|xy| \leq p$ and $y$ is not null,
  5. **You**: Need to choose an $i \geq 0$ such that $xy^iz$ is not in $L$ (in order to win (the prize of non-regularity)!).

- See how this works for showing $\{0^n1^m \mid n > m\}$ is not regular.
- Another example: Show $ADD = \{x=y+z \mid x, y, z$ are binary numbers and $x$ is the sum of $y$ and $z\}$ is not regular

---

Da Pumpin’ Lemma
(Lyrics: Harry Mairson)

Any regular language $L$ has a magic number $p$
And any long-enough word $s$ in $L$ has the following property:
Amongst its first $p$ symbols is a segment you can find
Whose repetition or omission leaves $s$ amongst its kind.

So if you find a language $L$ which fails this acid test,
And some long word you pump becomes distinct from all the rest,
By contradiction you have shown that language $L$ is not
A regular guy, resilient to the damage you have wrought.

But if, upon the other hand, $s$ stays within its $L$,
Then either $L$ is regular, or else you chose not well.
For $s$ is $xyz$, and $y$ cannot be null,
And $y$ must come before $p$ symbols have been read in full.

Source: [http://www.cs.brandeis.edu/~mairson/poems/node1.html](http://www.cs.brandeis.edu/~mairson/poems/node1.html)
If \( \{0^n1^n \mid n \geq 0\} \) is not Regular, what is it?

Irregular??

Enter…the world of Grammars