CSE 322
Winter Quarter 2001
Assignment 3
Due Friday, January 26

All solutions should be neatly written or typeset. All major steps in proofs and algorithms must be justified.

1. (10 points) For this problem you will practice converting a NFA to a DFA. Convert the following NFA to a DFA.
   Show only the reachable states of the DFA. The transition function should be given in a table.

2. (10 points) For this problem you will have practice in showing that regular languages are closed under more
   operations using finite automata contructions. We define the interleaving of two languages $A$ and $B$ over $\Sigma$ by
   
   \[ A \| B = \{ x_1 y_1 \cdots x_n y_n : x_i, y_i \in \Sigma^* , x_1, x_2, \cdots, x_n \in A, \text{ and } y_1, y_2, \cdots, y_n \in B \} \]

   For example if $A = \{ a, ab \}$ and $B = \{ 01 \}$ then $A \| B = \{ a01, 0a1, 01a, ab01, a0b1, a01b, 0ab1, 0a1b, 01ab \}$.
   Show that if $A$ and $B$ are regular then so is $A \| B$. Start with DFA's $M_1$ and $M_2$ that accept $A$ and $B$, respectively.
   Then construct an NFA that accepts $A \| B$.

3. (10 points) For this problem you should design algorithms in the style of problem 3 of the first assignment to
   decide properties of deterministic finite automata. In particular given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ there is a
   natural directed graph $G_M$ that models the transitions. The set of vertices of $G_M$ is $Q$ and $(q, p)$ is an edge in
   $G_M$ if $\delta(q, \sigma) = p$ for some $\sigma \in \Sigma$. The graph $G_M$ is the transition diagram of $M$ with the labels on the edges
   removed. You should use $G_M$ in your algorithms.

   (a) Design an algorithm to decide whether a DFA accepts any strings at all. That is, the algorithm given a
       finite automaton $M$ determines if $L(M) = \phi$.
   (b) Design an algorithm to decide whether a DFA accepts infinitely many strings.