1. Circle True or False below. Very briefly justify your answers, e.g. by giving a counter example, by citing a theorem we’ve proved, briefly sketching a construction, etc. Assume \( A \) and \( R \) are subsets of \( \Sigma^* \) for some fixed alphabet \( \Sigma \).

   (a) If \( R \) is regular, and \( A \subseteq R \), then \( A \) is regular. ................................. T  F

   (b) If \( R \) is regular, and \( R \subseteq A \), then \( A \) is regular. ................................. T  F

   (c) If \( R \) is regular, and \( A \cap R \) is regular, then \( A \) is regular. ............................. T  F

   (d) If \( R \) is regular, but \( A \cap R \) is non-regular, then \( A \) is non-regular. ...................... T  F

   (e) If \( R \) is regular, then \( R^* \) is regular. .............................................................. T  F
2. Give a deterministic finite automaton recognizing the language $L = \{x \in \{a, b\}^* \mid x \text{ contains an even number of } a\text{'s and an odd number of } b\text{'s}\}$. E.g., $b$ and $aaaba$ are in $L$, but $abab$ and $baaa$ are not. You do not need to give a correctness proof for your machine.

3. Consider the NFA $M = (Q, \Sigma, \delta, q_0, F)$ with the following transition diagram:

(a) In what states might the NFA be after reading input $bbba$? 
(b) Does the NFA accept $bbba$? Why or why not? 
(c) Suppose you apply the “subset” construction to build an equivalent DFA $M' = (Q', \Sigma, \delta', q'_0, F')$. What state $q \in Q'$ would $M'$ be in after reading the input $bbba$? 
(d) Is $q$ above in $F'$? Why or why not? 
(e) In terms of the states of $M$, what is the start state of $M'$? $q'_0 = $ 
(f) What state is $\delta'(\{2, 4\}, a)$? $\delta'(\{2, 6\}, a)$? $\delta'(\{5\}, a)$? 
(g) Describe in English the language accepted by $M$. (Say what it is, not how $M$ operates.)
4. Using the construction given in the text and lecture for converting an FA to a regular expression, eliminate state number 2 (and only state 2) from the following GNFA. The special start- and final-states have already been added. Arrows labeled $\emptyset$ are not shown. You may also omit them from your answer if you prefer, and you may simplify terms involving $\emptyset$ (e.g., $x \cup y \cdot \emptyset \equiv x$), but do not otherwise simplify the expressions.

5. Let $L = \{x \in \{a, b\}^* \mid x$ contains more a’s than b’s $\}$. Prove (using any method you wish) that $L$ is not a regular language.