1. Circle True or False below. *Very briefly justify your answers*, e.g. by giving a counter example, by citing a theorem we’ve proved, *briefly* sketching a construction, etc. Assume \( A \) and \( R \) are subsets of \( \Sigma^* \) for some fixed alphabet \( \Sigma \).

(a) If \( R \) is regular, and \( A \subseteq R \), then \( A \) is regular. ............................................... T F

| FALSE. Counterexample: \( A = \{a^n b^n \mid n \geq 0\} \), \( R = \{a, b\}^* \). |

(b) If \( R \) is regular, and \( R \subseteq A \), then \( A \) is regular. ............................................... T F

| FALSE. Counterexample: \( A = \{a^n b^n \mid n \geq 0\} \), \( R = \emptyset \). |

(c) If \( R \) is regular, and \( A \cap R \) is regular, then \( A \) is regular. ............................................... T F

| FALSE. Counterexample: \( A = \{a^n b^n \mid n \geq 0\} \), \( R = \emptyset \). |

(d) If \( R \) is regular, but \( A \cap R \) is non-regular, then \( A \) is non-regular. ............................................... T F

| TRUE, by closure of the class of regular languages under \( \cap \). |

(e) If \( R \) is regular, then \( R^* \) is regular. ............................................... T F

| TRUE, by closure of the class of regular languages under \( ^* \). |

2. Give a *deterministic* finite automaton recognizing the language \( L = \{x \in \{a, b\}^* \mid x \text{ contains an even number of } a\'s\text{ and an odd number of } b\'s\} \). E.g., \( b \) and \( aaaa \) are in \( L \), but \( aabab \) and \( baaa \) are not. You do not need to give a correctness proof for your machine.

![Diagram](image)

Note: State names indicate parity of \#a, \#b.

3. Consider the NFA \( M = (Q, \Sigma, \delta, q_0, F) \) with the following transition diagram:

![Diagram](image)
(a) In what states might the NFA be after reading input $bbba$?

(b) Does the NFA accept $bbba$? Why or why not? 

(c) Suppose you apply the “subset” construction to build an equivalent DFA $M' = (Q', \Sigma, \delta', q_0', F')$.

What state $q \in Q'$ would $M'$ be in after reading the input $bbba$?

(d) Is $q$ above in $F'$? Why or why not? 

(e) In terms of the states of $M$, what is the start state of $M'$? $q_0' = \{1, 2\}$

(f) What state is $\delta'(\{2, 4\}, a)$?

(g) Describe in English the language accepted by $M$. (Say what it is, not how $M$ operates.)

FYI, a corresponding regular expression would be $(bbba^*) \cup b^* (ab^* ab^*)^*$.

4. Using the construction given in the text and lecture for converting an FA to a regular expression, eliminate state number 2 (and only state 2) from the following GNFA. The special start- and final-states have already been added. Arrows labeled $\emptyset$ are not shown. You may also omit them from your answer if you prefer, and you may simplify terms involving $\emptyset$ (e.g., $x \cup y \cdot \emptyset \equiv x$), but do not otherwise simplify the expressions.

5. Let $L = \{x \in \{a, b\}^* \mid x$ contains more $a$’s than $b$’s $\}$. Prove (using any method you wish) that $L$ is not a regular language.

Assume $L$ is regular. Let $p$ be the pumping length for $L$, and let $s = a^p b^{p-1}$. Clearly $s$ has more $a$’s than $b$’s, so it is in $L$, and $|s| \geq p$, so by the pumping lemma there must exist strings $x, y, z$ such that $|xy| \leq p$, $|y| > 0$, and for all $i \geq 0$, $xy^i z \in L$. But $xy^0 z = a^p - |y| b^{p-1} \notin L$, since $p - |y| \leq p - 1$. This contradicts the conclusion of the pumping lemma, and therefore $L$ cannot be regular.