CSE 322 Lecture 3: Review of Proof Techniques

✦ Last Time:
  ✴ Proof by counterexample: Give an example that disproves the given statement
  ✴ Proof by contradiction: Assume statement is false and show that it leads to a contradiction
  ✴ Proof of set equality $A = B$: Show $A \subseteq B$ and $B \subseteq A$

✦ Today (and beyond):
  ✴ Proof of “X iff Y” statements
  ✴ Proof by construction
  ✴ Proof by induction
  ✴ “Bird-based” techniques: Pigeonhole principle and Dovetailing
  ✴ CS Theoretician’s favorite: Diagonalization

Proof Techniques II: The Big picture

✦ Proving “X iff Y” statements: Prove $X \Rightarrow Y$ (“X only if Y”) and $Y \Rightarrow X$ (“X if Y”)
  ✴ Example: For all real numbers $x$, show $\lfloor x \rfloor = \lceil x \rceil \iff x \in \mathbb{Z}$

✦ Proof by construction: Show that a statement can be satisfied by constructing an object using what is given
  ✴ Example: Show that for all $c$, there exists $n_0$ such that $n^2 > c$ for all $n \geq n_0$

✦ Proof by induction (very common in CS Theory): 2 steps –
  1. Basis Step: Show statement is true for some finite value $n_0$,
     typically $n_0 = 0$
  2. Induction hypothesis and induction step: Assume statement is true for some fixed but arbitrary $n \geq n_0$. Show it is also true for $n + 1$
  ✴ Example: Show that for all $n \geq 0$, $1 + 2 + \ldots + n = n(n+1)/2$

The “Avian” Techniques

✦ Pigeonhole principle: If A and B are finite sets and $|A| > |B|$, then there is no one-to-one function from A to B
  ✴ $f: A \rightarrow B$ is one-to-one if for any distinct $x, y \in A$, $f(x) \neq f(y)$
  ✴ Idea: “more pigeons than pigeonholes” ⇒ at least one pigeonhole contains two pigeons. Prove by induction on $|B|$
  ✴ E.g. In a room of 13 or more people, at least 2 have the same birthmonth

✦ Dovetailing: Useful for showing union of any finite or countably infinite collection of countably infinite sets is again countably infinite
  ✴ A is countably infinite if there is a 1-1 correspondence (“bijection”) between N (the set of natural numbers) and A
  ✴ E.g. Use dovetailing to show $\mathbb{Z}$ and $\mathbb{N} \times \mathbb{N}$ are both countably infinite

Next Class: Enter the finite automaton…

✦ Next time:
  ✴ Infinite sets that are not countably infinite (diagonalization)
  ✴ Finite automata 101

✦ Things to do over the weekend:
  ✴ Browse course website
  ✴ Sign up for mailing list (instructions on website)
  ✴ Finish Chapter 0 and start Chapter 1
  ✴ Start (and finish?) homework #1
  ✴ Have a great weekend!