CSE 321 Discrete Structures

January 29, 2010

Lecture 10: Program Correctness
Announcements

• Homework #4 will be posted tomorrow
  – Slightly shorter, because of the midterm

• Midterm: Friday in class (1:30-2:30)

• Makeup midterm: Wednesday, Feb. 3, CSE 503, 3:30pm-4:30pm
  – If you plan to take the midterm on Wednesday, please send me email in advance
  – (Backup: 4:30-5:30; hopefully we won’t need)
What to Study for the Midterm

• Read ALL lecture notes

• Read ALL handouts

• Review the homework solutions

• Rosen: read chapters 1, 4, 7.1, and 7.2
Binary Search

/* assume a[0] ≤ a[1] ≤ … ≤ a[n-1] */
/* find i in the array a[ ]: */
/* either find i such that a[i] = x */
/* or find i such that a[i] < x < a[i+1] */
Simplified Binary Search

We will assume first that $a[0] \leq x$

```c
int i = 0; int j = n;
while (i+1 < j) {
    int k = (i + j) / 2;
    if (x < a[k]) j = k;
    if (x >= a[k]) i = k;
}
```
Simplified Binary Search

Precondition:

\[ a[0] \leq x \land a[n] = \infty \land \forall u, v \in \{0..n-1\}. \ u < v \implies a[u] \leq a[v] \]

Postcondition:

\[ i+1=j \land a[i] \leq x < a[j] \]
Proof (Assuming \(a[0] \leq x\))

Loop invariant:

\[
(\text{precondition}) \land i < j \land a[i] \leq x < a[j]
\]

(Prove partial correctness on the white board)
Binary Search

Now we drop the assumption that $a[0] \leq x$

```c
int i = -1; int j = n;
while (i+1 < j) {
    int k = (i + j) / 2;
    if (x <= a[k]) j = k;
    if (x >= a[k]) i = k;
}
```
Binary Search

Precondition:

\[ a[-1] = -\infty \land a[n] = \infty \land \forall u, v \in \{0..n-1\}. u < v \Rightarrow a[u] \leq a[v] \]

Postcondition:

\[ (i+1=j \land a[i] < x < a[j]) \lor (i=j \land a[i] = x) \]
Proof

Loop invariant:

(precondition) \land
(i < j \land a[i] < x < a[j]) \lor (i = j \land a[i] = x)

a[ ] =

0 1 ... i j

x may be here

(Prove partial correctness on the white board)