Reading Assignment: Sections 4.1 – 4.3 and 5.1.

Problems:

1. Which amounts of money can be formed using just dimes and quarters? Prove your answer using a form of mathematical induction.

2. Assume that a chocolate bar consists of \( n \) squares arranged in a rectangular pattern. Any chocolate bar can be broken into two pieces along a horizontal or vertical line separating the squares (e.g. a \( 2 \times 4 \) chocolate bar can be broken in 4 different ways: along the one horizontal line, or along one of the three vertical lines). Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into \( n \) separate squares. Use strong induction to prove your answer.

3. Section 4.2, exercise 12.

4. Give a recursive definition of
   
   (a) the set of odd positive integers,
   
   (b) the set of positive integer powers of 3,
   
   (c) the set of positive integers congruent to 4 modulo 5.

5. Let \( f_n \) be the \( n \)-th Fibonacci number. Prove that \( f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1} \) whenever \( n \) is a positive integer.

6. Structural induction: Show that the set \( S \) defined by \( 1 \in S \) and \( s + t \in S \) whenever \( s \in S \) and \( t \in S \) is the set of positive integers.
   (Hint: You are showing that two sets \( A \) and \( B \) are equal, which requires showing that both \( A \subseteq B \) and \( B \subseteq A \).)