Discrete Structures

Functions

Chapter 2, Section 2.3

Dieter Fox
Functions

♦ \( f : A \rightarrow B \): A function from \( A \) to \( B \) is an assignment of exactly one element of \( B \) to each element of \( A \).

♦ \( A \) is the domain of \( f \) and \( B \) is the codomain of \( f \).

♦ If \( f(a) = b \), we say that \( b \) is the image of \( a \) and \( a \) is a pre-image of \( b \). The range of \( f \) is the set of all images of elements of \( A \).

♦ \( f \) maps from \( A \) to \( B \).

♦ \( f_1 + f_2, f_1 f_2 \): Let \( f_1 \) and \( f_2 \) be functions from \( A \) to \( R \). Then
  \[
  (f_1 + f_2)(x) = f_1(x) + f_2(x),
  \]
  \[
  (f_1 f_2)(x) = f_1(x) f_2(x)
  \]
Functions

♦ **Injection:** Function $f$ is said to be one-to-one, if and only if $f(x) = f(y)$ implies that $x = y$ for all $x$ and $y$ in the domain of $f$.

♦ Function $f$ whose domain and codomain are subsets of the set of real numbers is called strictly increasing if $f(x) < f(y)$ whenever $x < y$ and $x$ and $y$ are in the domain of $f$ (decreasing analogous).

♦ **Surjection:** Function $f$ is said to be onto / surjective, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

♦ **Bijection:** Function $f$ is a one-to-one correspondence, or bijection, if it is both one-to-one and onto.

♦ **Inverse function:** Let $f$ be a one-to-one correspondence from $A$ to $B$. The inverse function of $f$ assigns to an element $b$ in $B$ the unique element $a$ in $A$ such that $f(a) = b$. The inverse function of $f$ is denoted by $f^{-1}$. Hence, $f^{-1}(b) = a$ when $f(a) = b$. 
Functions

♦ $f \circ g$: $g : A \to B$, $f : B \to C$. The composition of the functions $f$ and $g$ is defined by
\[(f \circ g)(a) = f(g(a))\]

♦ $\lfloor x \rfloor$ The floor function assigns to the real number $x$ the largest integer that is less than or equal to $x$.

♦ $\lceil x \rceil$ The ceiling function assigns to the real number $x$ the smallest integer that is greater than or equal to $x$. 