**Insertion Sort**

A: 0 .. n-1
For i = 1 .. n-1 {
    T = A[i]
    j = i
    while j > 0 & T < A[j-1] {
        j = j - 1
    }
    A[j+1] = T
Run Time
worst-case $O(n^2)$ \( \approx \frac{n^2}{2} \) swaps

- compare = \# swap + \( n-1 \)

\( n! \) "different" inputs:
- Assume all \( n! \) inputs equally likely

Permutations:

\[
\begin{array}{c}
1 & 2 & 3 & 4 & 5 \\
1 & 3 & 5 & 2 & 4
\end{array}
\]

\( (i,j) \) is an inversion in \( \pi \)
- if \( i < j \) but \( \pi_i > \pi_j \)

- Example:
  - \( 12345 \) : No inversion
  - \( 54321 \) : \( \frac{5}{2} \) inversions
    (The max)

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Swapping an adjacent pair of positions that are out of order decreases # of inversions by exactly 1.

# of swaps by insertion sort exactly # of inversions in its input

\[ I_{i,j} = \begin{cases} 
1 & \text{if } (i,j) \text{ is an inversion} \\
0 & \text{if not} 
\end{cases} \]

\[ \# \text{ of inversions } \quad I = \sum_{i<j} I_{i,j} \]
\[ E(I) = \sum_{i<j} E(I_{ij}) \]

\[ \pi \]

\[ \pi' \]

\[ \phi(I_{ij} = 1) = E(I_{ij}) = \frac{1}{2} \]

(for every \( \pi \) where \((i,j)\) is an inversion, there is \( \pi' \) where it's not.)

\[ E(I) = \sum_{ij} \pi_{ij} \cdot \frac{1}{2} = \left(\frac{n}{2}\right) \cdot \frac{1}{2} \]

\[ \therefore \text{Expected \# of swaps} = \left(\frac{n}{2}\right)/2 \]

vs worst case \( (\frac{\binom{n}{2}}{2}) \)

i.e. average runtime (assuming random input) is \( \sim \frac{1}{2} \) of worst case
\[E(x+y) = E(x) + E(y)\]

\[\forall (x+y) = \forall (x) + \forall (y) \quad \text{(in general)}\]

but = if \(x\) \& \(y\) are indp.

\[E(x \cdot y) \neq E(x) \cdot E(y)\]

\(\neq\) in general, but

\[E(x \cdot y) \neq E(x) \cdot E(y)\]

Example:

\(x = 0/1\) and \(w / p = 1/2\)

\(y = x\)

\[E(x) = \frac{1}{2} = E(y)\]

\[E(x \cdot y) = \frac{1}{2} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 0 \cdot 0 = \frac{1}{2} \neq \left(\frac{1}{2}\right)^2 = E(x) \cdot E(y)\]