321 Section

Natalie Linnell
The importance of domains

- Domain is part of the function definition
- A predicate is a function from some domain to \{T,F\}
- If \( P(x) \) means that \( x \) is odd, the domain can’t be the real numbers
∀ x (P(x) ∨ Q(x))
∀ x P(x) ∨ ∀ x Q(x)
Artificially restricting the domain

- There is someone who has visited every country except for Libya.
\[ \exists x \ \forall y (\neg \text{InWith}(x, \text{Kevin}) \land \neg \text{InWith}(x, y) \lor \neg \text{InWith}(y, \text{Kevin})) \]

• How do you translate this into English?
∀ x (P(x) → Q(x))
∃ y (P(x) ∧ Q(x))
Using constants

• There is someone in the class who speaks Hindi.
HW3, Problem 2
Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.
Existence proofs

(it’s ok to just give an example!)
\[ \exists A \forall B \quad Q(A, B) \]
\[ \exists B \forall A \quad Q(A, B) \]

Q(A, B) iff \( A \subseteq B \)
Is symmetric difference associative?
What does Fermat’s Little Theorem mean?

- \( a^{p-1} \mod p = 1 \) if \( p \) is prime
- Can I use it to solve the following, and if so what’s the answer?
  - What’s \( 2^4 \mod 4 \)?
  - What’s \( 2^3 \mod 5 \)?
  - What’s \( 2^4 \mod 5 \)?
  - What’s \( 2^{442} \mod 5 \)?
Prove that if \( m \) and \( n \) are both perfect squares, then \( mn \) is a perfect square.

- What kind of proof did you do?
Prove that if $3n+2$ is odd, then $n$ is odd

• What kind of proof did you do?
Show that the statement “Every positive integer is the sum of the squares of two integers” is false