Announcements

- Readings
  - Counting
    • 5.5, (4.5) Generalized Permutations and Combinations
  - Probability Theory
    • 6.1, 6.2 (5.1, 5.2) Probability Theory
    • 6.3 (New material!) Bayes’ Theorem
    • 6.4 (5.3) Expectation
  - Advanced Counting Techniques – Ch 7.
    • Not covered

Highlights from Lecture 17

- Permutations
  \[ P(n, r) = \frac{n!}{(n-r)!} \]
- Combinations
  \[ C(n, r) = \binom{n}{r} = \frac{n!}{(n-r)!r!} \]

How many

- Let \( s_1 \) be a string of length \( n \) over \( \Sigma_1 \)
- Let \( s_2 \) be a string of length \( m \) over \( \Sigma_2 \)
- Assuming \( \Sigma_1 \) and \( \Sigma_2 \) are distinct, how many interleavings are there of \( s_1 \) and \( s_2 \)?

Permutations with repetition

Combinations with repetition

- How many different ways are there of selecting 5 letters from \( \{A, B, C\} \) with repetition
How many non-decreasing sequences of \{1,2,3\} of length 5 are there?

- 1 + 2 + 2
- 2 + 0 + 3
- 0 + 1 + 4
- 3 + 1 + 1
- 5 + 0 + 0

How many different ways are there of adding 3 non-negative integers together to get 5?

- 1 + 2 + 2
- 2 + 0 + 3
- 0 + 1 + 4
- 3 + 1 + 1
- 5 + 0 + 0

C(n+r-1,n-1) r-combinations of an n element set with repetition

Permutations of indistinguishable objects

- How many different strings can be made from reordering the letters ABCDEFGH
- How many different strings can be made from reordering the letters AAAABBBB
- How many different strings can be made from reordering the letters GOOOGLE

Discrete Probability

- **Experiment**: Procedure that yields an outcome
- **Sample space**: Set of all possible outcomes
- **Event**: subset of the sample space

S a sample space of equally likely outcomes, E an event, the probability of E, p(E) = |E|/|S|

Example: Dice
Example: Poker
Probability of 4 of a kind

Combinations of Events

$E^c$ is the complement of $E$

$$P(E^c) = 1 - P(E)$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$