Announcements

• Reading for this week
  – Today: 2.1, 2.2 (5th Edition: 1.6, 1.7)
  – Thursday: 2.3 (5th Edition: 1.8)
  – Friday: 3.4, 3.5 (5th Edition: 2.4, 2.5)

• Homework 3
  – Due Wednesday, January 30
  – Note: problems are not necessarily of the same degree of difficulty

Highlights from Lecture 6

• Direct Proofs
• Chomp!
• Challenges
  – Develop optimal strategy for 6 × 8 Chomp!
  – Create a Chomp! program that uses an optimal algorithm
  – Generalizations of Chomp!

Set Theory

• Formal treatment dates from late 19th century
• Direct ties between set theory and logic
• Important foundational language

Definition: A set is an unordered collection of objects

Definitions

• A and B are equal if they have the same elements
  \[ A = B \iff \forall x \ (x \in A \iff x \in B) \]

• A is a subset of B if every element of A is also in B
  \[ A \subseteq B \iff \forall x \ (x \in A \rightarrow x \in B) \]
Empty Set and Power Set

Cartesian Product: $A \times B$

\[ A \times B = \{ (a, b) \mid a \in A \land b \in B \} \]

Set operations

\[
\begin{align*}
A \cup B &= \{ x \mid x \in A \lor x \in B \} \\
A \cap B &= \{ x \mid x \in A \land x \in B \} \\
A - B &= \{ x \mid x \in A \land x \notin B \} \\
A \oplus B &= \{ x \mid x \in A \oplus x \in B \} \\
\overline{A} &= \{ x \mid x \notin A \}
\end{align*}
\]

De Morgan’s Laws

\[
\begin{align*}
\overline{A \cup B} &= \overline{A} \cap \overline{B} \\
\overline{A \cap B} &= \overline{A} \cup \overline{B}
\end{align*}
\]

Proof technique:
To show $C = D$ show
$x \in C \implies x \in D$ and
$x \in D \implies x \in C$

Distributive Laws

\[
\begin{align*}
A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\
A \cup (B \cap C) &= (A \cup B) \cap (A \cup C)
\end{align*}
\]

Russell’s Paradox

\[ S = \{ x \mid x \notin x \} \]