Announcements

• Reading for Monday: 1.3
• Signup for the mailing list
  – See course website for details
• Office hours
  – Richard Anderson, CSE 582, Friday 2:30-3:30
  – Natalie Linnell, CSE 218, Monday, 11:00-12:00, Tuesday, 2:00-3:00

Highlights from Lecture 2

• Tautology: A compound proposition that is always true.
• P is equivalent to Q (P ≡ Q): P ↔ Q is a tautology
• Truth table algorithm for testing equivalence
• DeMorgan’s laws
  – ¬ (p ∨ q) = __________
  – ¬ (p ∧ q) = __________

Equivalences relating to implication

• p → q ≡ ¬ p ∨ q
• p → q ≡ ¬ q → ¬ p
• p ∨ q ≡ ¬ p → q
• p ∧ q ≡ ¬ (p → ¬ q)
• p ↔ q ≡ (p → q) ∧ (q → p)
• p ↔ q ≡ ¬ p ↔ ¬ q
• p ↔ q ≡ (p ∧ q) ∨ (¬ p ∧ ¬ q)
• ¬ (p ↔ q) = p ↔ ¬ q

Logical Proofs

• To show P is equivalent to Q
  – Apply a series of logical equivalences to subexpressions to convert P to Q
• To show P is a tautology
  – Apply a series of logical equivalences to subexpressions to convert P to T

Why bother with logical proofs when we have truth tables?
Show \((p \land q) \rightarrow (p \lor q)\) is a tautology

Show \((p \rightarrow q) \rightarrow r\) and \(p \rightarrow (q \rightarrow r)\) are not equivalent

Predicate Calculus

- Predicate or Propositional Function
  - A function that returns a truth value
- "x is a cat"
- "x is prime"
- "student x has taken course y" 
- "x > y"
- "x + y = z"

Statements with quantifiers

- \(\exists x \text{ Even}(x)\)
- \(\forall x \text{ Odd}(x)\)
- \(\forall x (\text{Even}(x) \lor \text{Odd}(x))\)
- \(\exists x (\text{Even}(x) \land \text{Odd}(x))\)
- \(\forall x \text{ Greater}(x+1, x)\)
- \(\exists x (\text{Even}(x) \land \text{Prime}(x))\)

Quantifiers

- \(\forall x P(x) : P(x)\) is true for every \(x\) in the domain
- \(\exists x P(x) :\) There is an \(x\) in the domain for which \(P(x)\) is true

Statements with quantifiers

- \(\forall x \exists y \text{ Greater}(y, x)\)
- \(\forall x \exists y \text{ Greater}(x, y)\)
- \(\forall x \exists y (\text{Greater}(y, x) \land \text{Prime}(y))\)
- \(\forall x (\text{Prime}(x) \rightarrow (\text{Equal}(x, 2) \lor \text{Odd}(x)))\)
- \(\exists x \exists y (\text{Equal}(x, y + 2) \land \text{Prime}(x) \land \text{Prime}(y))\)
Statements with quantifiers

- "There is an odd prime"
- "If x is greater than two, x is not an even prime"
- \( \forall x \forall y \forall z ((\text{Equal}(z, x+y) \land \text{Odd}(x) \land \text{Odd}(y)) \rightarrow \text{Even}(z)) \)
- "There exists an odd integer that is the sum of two primes"

Goldbach’s Conjecture

- Every even integer greater than two can be expressed as the sum of two primes