Midterm Exam, Friday, February 8, 2008

NAME:__________________________

Instructions:

• Closed book, closed notes, no cell phones, no calculators.

• Time limit: 50 minutes.

• Answer the problems on the exam paper.

• If you need extra space use the back of a page.

• Problems are not of equal difficulty, if you get stuck on a problem, move on.

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Problem 1 (10 points):

a) Show that the expression \((p \rightarrow q) \rightarrow (p \rightarrow r)\) is a contingency.

b) Give an expression that is logically equivalent to \((p \rightarrow q) \rightarrow (p \rightarrow r)\) using the logical operators \(\neg, \lor,\) and \(\land\) (but not \(\rightarrow\)).

Problem 2 (20 points):
Using the predicates:
\[ \text{Likes}(p, f)\]: “Person \(p\) likes to eat the food \(f\).”
\[ \text{Serves}(r, f)\]: “Restaurant \(r\) serves the food \(f\).”
translate the following statements into logical expressions.

a) Every restaurant serves a food that no one likes.

b) Every restaurant that serves TOFU also serves a food which RANDY does not like.

Translate the following logical expressions into English. (You may want to give a couple of sentences of explanation - the point of this question is to demonstrate that you understand the logical expression.)

c) \(\exists r \forall p \exists f (\text{Serves}(r, f) \land \text{Likes}(p, f))\)

d) \(\forall r \exists p \forall f (\text{Serves}(r, f) \rightarrow \text{Likes}(p, f))\)
Problem 3 (10 points):
Determine the value of the following. (You will probably want to use different methods to compute the values.)

a) \(3^{303} \mod 101\)

b) \(3^{64} \mod 100\)

Problem 4 (15 points):

a) What is public key cryptography?

b) What is the computation that “Alice” employs when she encodes a message using RSA?

c) Why is RSA considered secure?
Problem 5 (10 points):
Use rules of inference to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\neg R(a)$, where $a$ is in the domain, are true, then the conclusion $\neg P(a)$ is true. (Note: You do not need to give the names for the rules of inference.)

Problem 6 (10 points):
Prove that if $n$ is even and $m$ is odd, then $(n + 1)(m + 1)$ is even.

Problem 7 (10 points):
Prove or disprove:

a) For positive integers $x$, $p$, and $q$, $(x \mod p) \mod q = x \mod pq$.

b) For positive integers $x$, $p$, and $q$, $(x \mod p) \mod q = (x \mod q) \mod p$. 