CSE 321: Discrete Structures Spring 2007

Problem Set 5

Due Friday, May 11, 2007, in class

Instructions: Same as for Problem Set 1.

The exercise numbers refer to the number in Rosen's book, 6th Edition. When a different number is used in the 5th edition, that number is also mentioned.

Reading Sections: 6th Edition: 4.3,5.1-5.4. 5th edition: 3.4,4.1-4.4.

1. (16 points)

   (a) Section 4.3, Exercise 36. (5th Edition: Section 3.4, Exercise 36)

   (b) Section 4.3, Exercise 44. (5th Edition: Section 3.4, Exercise 44)

2. (20 points) Solve the following counting problems. In each case show the reasoning that leads you to your answer.

   (a) A palindrome is a word that reads the same forwards and backwards. How many seven-letter palindromes can be made from the English alphabet?

   (b) A professor writes 20 discrete mathematics true/false questions. Of the statements in these questions, 13 are true. If the questions can be positioned in any order, how many different answer keys are possible?

   (c) Suppose you have n beads, each of a different color, that you need to string into a necklace? How many distinct necklaces can you make? (A necklace flipped over remains the same and does not count as a distinct necklace.)

   (d) How many 5 card hands from a 52 card deck have at least one card from each of the four different suits (Diamond, Hearts, Clubs, and Spades)?

   (e) How many ways are there for a horse race with four horses to finish if ties are possible? (Any number of the four horses may tie.)

3. ($8 + 10 = 18$ points)

   (a) Imagine a town with East-West streets numbered 1 through n and North-South Avenues numbered 1 through m. A taxi cab picks you up at the corner of 1st Avenue and 1st street, and you wish to be dropped off at the corner of m'th avenue and n'th street. Since you are a smart 321 student, you are obviously not going to permit the cab driver to drive longer than the necessary $(n-1)+(m-1)$ blocks. In other words, the cabby must always be increasing his street number or his avenue number. Suppose there is an accident at the intersection of street i and avenue j, for some i, j where $1 < i < n$ and $1 < j < m$. How many routes can the cab driver take to get you to your destination while avoiding the intersection with the accident? Justify your answer.

   (b) (Could be a little tricky and tedious; attempt after the rest of the questions)
   A popular award-winning game called Set (`www.setgame.com`) is based on cards with the following four features:

Color: Each card is red, green, or purple.

Symbol: Each card contains ovals, squiggles, or diamonds.

Number: Each card has one, two, or three symbols.

Shading: Each card is solid, open, or striped.

So in all the game consists of $3^4 = 81$ distinct cards.

A 'Set' consists of three distinct cards in which each feature is *either the same on each card or is different on each card.* In other words, any feature in the 'Set' of three cards is either common to all three cards or is different on each card.

In this exercise, your task is to count the number of distinct such 'Sets'. (Note that the ordering of the cards within a 'Set' doesn't matter, but just which three cards belong to the 'Set'.)

4. (16 points)

   (a) In a dinner party with n people, all of them are seated at a circular table. Suppose there is a name tag at each place at the table and suppose that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

   (b) Prove that every set of ten distinct numbers between 1 and 100 contains two disjoint nonempty subsets with the same sum.