Reading Assignment: Section 7.1-7.5 and 8.1-8.5 of Rosen.

Problems: (Note: For probability problems, please describe the process of how to get the answer.)

1. Section 5.1, exercise 40.

2. Section 5.2, exercise 28, 30.

3. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

4. Suppose a 6-sided fair dice is rolled. Let the random variable $X$ be the value showing. What is the expectation of $X$? Suppose two fair 6-sided dice are rolled independently. Let $Y$ be the random variable which is the sum of the two values showing. What is the expectation of $Y$? Let $Z$ be the random variable which is the minimum of the two values showing. What is the expected value of $Z$?

5. Suppose that a fair coin is tossed 1000 times. Let $X$ be the random variable which is the number of flips $i$ in which the coin takes the same value in both flip $i$ and $i + 1$. What is the expected value of $X$? (For example, in the sequence $HHHH$, $X$ is 3, and in the sequence $TTHHHTT$, $X$ is also 3.)

6. Let $E, F$ be events with $P(F) \neq 0$. Prove that
   \[ P(E) = P(E|F)P(F) + P(E|\bar{F})P(\bar{F}). \]

7. Section 5.3, exercise 10, 16.

8. Section 7.1, exercise 4.