Outline

◊ Propositional Logic

◊ Propositional Equivalences

◊ First-order Logic
Propositional Logic

Let \( p \) and \( q \) be propositions.

- **Negation** \( \neg p \)  The statement “It is not the case that \( p \)” is true, whenever \( p \) is false and is false otherwise.

- **Conjunction** \( p \land q \)  The statement “\( p \) and \( q \)” is true when both \( p \) and \( q \) are true and is false otherwise.

- **Disjunction** \( p \lor q \)  The statement “\( p \) or \( q \)” is false when both \( p \) and \( q \) are false and is true otherwise.

- **Exclusive or** \( p \oplus q \)  The *exclusive or* of \( p \) and \( q \) is true when exactly one of \( p \) and \( q \) is true and is false otherwise.
Propositional Logic

Let $p$ and $q$ be propositions.

◊ **Implication** $p \rightarrow q$ The *implication* $p \rightarrow q$ is false when $p$ is true and $q$ is false and is true otherwise. $p$ is called the *hypothesis* (antecedent, premise) and $q$ is called the *conclusion* (consequence).

• “if $p$, then $q$" "$p$ implies $q$" "$p$ only if $q$" "$p$ is sufficient for $q$" "$q$ is necessary for $p$"

• $q \rightarrow p$ is called the *converse* of $p \rightarrow q$

• $\neg q \rightarrow \neg p$ is called the *contrapositive* of $p \rightarrow q$

◊ **Biconditional** $p \leftrightarrow q$ The *biconditional* $p \leftrightarrow q$ is true whenever $p$ and $q$ have the same truth values and is false otherwise.
Translating English Sentences

◊ You can access the Internet from campus only if you are a computer science major or you are not a freshman.

◊ You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.
Logical Equivalences

♦ **Tautology**  A compound statement that is always true.

♦ **Contradiction**  A compound statement that is always false.

♦ **Contingency**  A compound statement that is neither a tautology nor a contradiction.

♦ **Logical equivalence**  $p \equiv q$  Propositions $p$ and $q$ are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.
I don’t jump off the Empire State Building implies if I jump off the Empire State Building then I float safely to the ground.

\[(\text{Smoke} \land \text{Heat}) \to \text{Fire} \equiv ((\text{Smoke} \to \text{Fire}) \lor (\text{Heat} \to \text{Fire}))\]
# Logical Equivalences

<table>
<thead>
<tr>
<th>Identity laws</th>
<th>Domination laws</th>
<th>Idempotent laws</th>
<th>Double negation law</th>
<th>Commutative laws</th>
<th>Associative laws</th>
<th>Distributive laws</th>
<th>De Morgan’s laws</th>
<th>Absorption laws</th>
<th>Negation laws</th>
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<tbody>
<tr>
<td>$p \land T \equiv p$</td>
<td>$p \lor T \equiv T$</td>
<td>$p \lor F \equiv p$</td>
<td>$p \land F \equiv F$</td>
<td>$\neg (\neg p) \equiv p$</td>
<td>$p \lor q \equiv q \lor p$</td>
<td>$p \land q \equiv q \land p$</td>
<td>$(p \lor q) \lor r \equiv p \lor (q \lor r)$</td>
<td>$(p \land q) \land r \equiv p \land (q \land r)$</td>
<td>$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$</td>
</tr>
</tbody>
</table>

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First-order Logic

◊ **Universal quantifier** $\forall$: The *universal quantification* of $P(x)$ is the proposition “$P(x)$ is true for all values of $x$ in the universe of discourse.”

◊ **Existential quantifier** $\exists$: The *existential quantification* of $P(x)$ is the proposition “There exists an element $x$ in the universe of discourse such that $P(x)$ is true.”