Probability Theory

Let \( S \) be the sample space of an experiment with a finite or countable number of outcomes. We assign probability \( P(\omega) \) to each outcome \( \omega \). The following two conditions have to be met:

(i) For each \( \omega \in S \), \( 0 \leq P(\omega) \leq 1 \).
(ii) \( \sum_{\omega \in S} P(\omega) = 1 \).

The probability of the event \( E \) is the sum of the probabilities of the outcomes in \( E \). That is:

\[
\text{The probability of the event } E \text{ is the sum of the probabilities of the outcomes in } E.
\]

\[
\text{Let } E \text{ and } F \text{ be events with } P(F) > 0. \text{ The conditional probability of } E \text{ given } F \text{ is defined as }
\]

\[
P(E | F) = \frac{P(E \cap F)}{P(F)}
\]

The events \( E \) and \( F \) are said to be independent if and only if:

\[
P(E \cap F) = P(E)P(F)
\]

If \( E \) is an event in a sample space \( S \), then:

\[
P(E) = \sum_{\omega \in E} P(\omega)
\]

Let \( E \) and \( F \) be events in a sample space \( S \). Then:

\[
P(E \cup F) = P(E) + P(F) - P(E \cap F)
\]

Theorem: Let \( E \) and \( F \) be events in a sample space \( S \). The conditional probability of the event \( E \) given the event \( F \) is defined by:

\[
P(E | F) = \frac{P(E \cap F)}{P(F)}
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E | F) = P(E)
\]

\[
P(F | E) = P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E \cap F) = P(E)P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E | F) = P(E)
\]

\[
P(F | E) = P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E \cap F) = P(E)P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E | F) = P(E)
\]

\[
P(F | E) = P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E \cap F) = P(E)P(F)
\]

The events \( E \) and \( F \) are independent if and only if:

\[
P(E | F) = P(E)
\]

\[
P(F | E) = P(F)
\]
Then \( (X + \cdots + Z)A + (Y + \cdots + Z)A + (X + \cdots + Y + \cdots + Z)A \) = \( (X + \cdots + Z)A + (X + \cdots + Z)A + (X + \cdots + Z)A \) = \( X + \cdots + Z \)

Then \( (X + \cdots + Z)A = X + \cdots + Z \)

Moreover, if \( a \) and \( b \) are real numbers, then \( aA + bA = (a + b)A \) and \( a(\frac{1}{X + \cdots + Z})A = (a + b)A \)

Theorem: \( X \) and \( Y \) are independent random variables on a space \( S \).

The standard deviation of \( X \), denoted \( \sigma_X \), is defined to be \( \sigma_X = \sqrt{\mathbb{E}[(X - \mathbb{E}[X])^2]} \)

The variance of \( X \) is denoted by \( \text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] \)

Let \( X \) be random variables on a sample space \( S \). The variance of \( X \) is denoted by \( \text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] \)

The random variables \( X \) and \( Y \) on a sample space \( S \) are independent if for all real numbers \( a \) and \( b \):

\( \mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y] \)

The expected value (or expectation) of a random variable \( X(s) \) on the sample space \( S \) is equal to the set of real numbers:

A random variable is a function from the sample space of an experiment to the set of real numbers.

**Random Variables**

**Independence**

**Variance**

**Bernoulli Trial**

**Bernoulli Trials**

**Probability of success and probability of failure of Bernoulli trials**

**Bernoulli Trial:** Experiment with only two possible outcomes:

- **Success or failure.**