A permutation of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of $r$ elements of a set is called an $r$-permutation.

**Theorem:** The number of $r$-permutations of a set with $n$ elements, where $n$ is a positive integer and $r$ is an integer with $0 \leq r \leq n$, is

$$P(n, r) = \frac{n!}{(n-r)!}$$

where $n$ is a positive integer and $r$ is an integer with $0 \leq r \leq n$.

In how many ways can 20 students out of a class of 32 be chosen to attend a class on a late Thursday afternoon (and take notes for the others)? If no two children are next to each other.

In how many ways can ten adults and five children stand in a line so that no two children are next to each other?

In how many ways can ten adults and five children stand in a line so that no two children are next to each other?

**Examples**

**Combinations**

The number of $r$-combinations of a set with $n$ elements, where $n$ is an unordered selection of $r$ elements from the set.

**Theorem:** The number of $r$-combinations of a set with $n$ elements, where $n$ is an unordered selection of $r$ elements from the set.

**Examples**

**Permutations**

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$$P(n, r) = \frac{n!}{(n-r)!}$$

where $n$ is a positive integer and $r$ is an integer with $0 \leq r \leq n$.

6. Paul and Michelle refuse to attend class together.
5. Jim and Michelle attended.
4. Either Jim or Michelle (or both) go to class?
3. Jim and Michelle insist on going.
2. Michelle insists on going.
1. Paul refuses to go to class.

In how many ways can 20 students out of a class of 32 be chosen to attend a class on a late Thursday afternoon (and take notes for the others)? If no two children are next to each other.

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Pascal's Identity: Let $K$ and $k$ be positive integers with $K \leq k$. Then

\[ \begin{align*}
\binom{u}{u} + u \binom{1-u}{u} + \cdots + u^k \binom{1}{u} + u^0 \binom{0}{u} &= \\
\sum_{i=0}^{k} u^i \binom{k-1+i}{k-1} &= u^{k+1}
\end{align*} \]

Binomial Theorem: Let $x$ and $y$ be variables and let $n$ be a positive integer. Then

\[ (x+y)^n = \binom{n}{k} x^k y^{n-k} \]

Pascal's Identity: Let $u$ and $\gamma$ be positive integers with $u \leq \gamma$. Then

\[ \binom{u+\gamma}{u} \cdot \frac{1}{u!} = \binom{\gamma+1}{u+1} \cdot \frac{1}{(\gamma+1)!} \]