Basic Notions of Probabilistic Reasoning

$p(A)$

is the **probability** that proposition $A$ is true.

$p(L = \langle x, y, \theta \rangle) = 0.1$

the **probability** of the robot **being at location** \(<x, y, \theta>\) **given no other information** is 0.1

$L$

is denoted as a **random variable**
Probability Distributions

- Random variables can have multiple values:
  - boolean: True, False
  - multi-valued: high, low, medium
  - continuous

- A probability distribution specifies the probabilities for all possible values of a random variable.
Discrete vs. Continuous

Discrete:

\[ P(\text{Room}) = \langle 0.7, 0.2, 0.08, 0.02 \rangle \]

Continuous:
Conditional Probability

n The term $p(A)$ is a **prior probability**

n Suppose we also know $B$

n We use

$$p(A|B)$$

to describe the **probability of $A$ given** all we know is $B$. 
The Product Rule

Product rule:

\[ p(A \land B) = p(A \mid B) p(B) \]

Conditional probabilities can be expressed in terms of unconditional probabilities!
\[
\frac{(B)d}{(\forall)d\ (\forall|B)d} = (B|\forall)d
\]

\[
(\forall)d(\forall|B)d = (B)d(B|\forall)d = (B \lor \forall)d
\]
A Simple Example: State Estimation

- Suppose a robot obtains measurement $s$
- What is $p(\text{doorOpen}|s)$?
Causal vs. Diagnostic Reasoning

- \( p(\text{open}|s) \) is diagnostic
- Often causal knowledge like

\[
p(s|\text{open})
\]

is easier to obtain.

- Application of Bayes rule:

\[
p(\text{open}|s) = \frac{p(s|\text{open})p(\text{open})}{p(s)}
\]
Normalization

\[
p(\text{open} \mid s) = \frac{p(s \mid \text{open}) p(\text{open})}{p(s)}
\]

\[
p(\neg \text{open} \mid s) = \frac{p(s \mid \neg \text{open}) p(\neg \text{open})}{p(s)}
\]

\[
p(\text{open} \mid s) + p(\neg \text{open} \mid s) = 1
\]

\[\Rightarrow\]

\[
p(s) = p(s \mid \text{open}) p(\text{open}) + p(s \mid \neg \text{open}) p(\neg \text{open})
\]

\[\Rightarrow\]

\[
p(\text{open} \mid s) = \frac{p(s \mid \text{open}) p(\text{open})}{p(s \mid \text{open}) p(\text{open}) + p(s \mid \neg \text{open}) p(\neg \text{open})}
\]
Example

\[ p(s|\text{open}) = 0.6 \quad p(s|\neg\text{open}) = 0.3 \]
\[ p(\text{open}) = p(\neg\text{open}) = 0.5 \]

\[
p(\text{open} | s) = \frac{p(s | \text{open}) p(\text{open})}{p(s | \text{open}) p(\text{open}) + p(s | \neg\text{open}) p(\neg\text{open})}
\]

\[
p(\text{open} | s) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67
\]

\( s \) raises the probability, that the door is open.
Integrating a second Measurement ...

n New measurement $s_2$

n $p(s_2|\text{open}) = 0.5 \quad p(s_2|\neg\text{open}) = 0.6$

$$p(\text{open}|s_2s_1) = \frac{p(s_2|\text{open})p(\text{open}|s_1)}{p(s_2|\text{open})p(\text{open}|s_1) + p(s_2|\neg\text{open})p(\neg\text{open}|s_1)}$$

$$p(\text{open}|s_2s_1) = \frac{1 \cdot \frac{2}{2} \cdot \frac{3}{3}}{1 \cdot \frac{2}{2} + \frac{3}{3} \cdot \frac{1}{5} \cdot \frac{3}{3}} = \frac{5}{8} = 0.625$$

$s_2$ lowers the probability, that the door is open.