Section 6

continuous random variables :)

-Announcements-

Additional OH this Friday, Saturday, and Sunday for the Midterm

Review session: Friday 5-6pm G01

See Ed for more details

-Schedule-

2/5	2/6 Pset 4 grades released	2/7 Pset 5 due	2/8 Section	2/9 Midterm review Pset 5 Task 7 due
2/12 Midterm	2/13	2/14 Pset 6 released	2/15	2/16

Discrete vs Continuous Random var.

discrete

the range consists of finite/countably infinite values

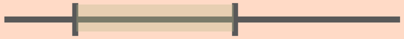
two “types” of random vars

discrete

the range consists of finite/countably infinite values

two "types" of random vars

continuous



the range consists of uncountably infinite values (*for example time is not discrete)*

discrete

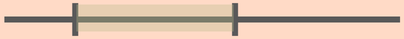
the range consists of finite/countably infinite values

PMF (prob. mass function)

$$p_X(k) = P(X=k)$$

two “types” of random vars

continuous



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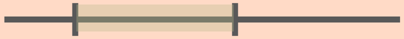

the range consists of uncountably infinite values (*for example time is not discrete)*

PMF (prob. mass function)

$p_X(k) = P(X=k) = 0$

two “types” of random vars

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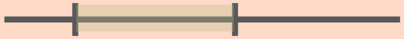
PMF (prob. mass function)

$p_x(k) = P(X=k)$

continuous



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PDF (prob. ***density*** *function)*

$f_x(k)$!= $P(X=k)$

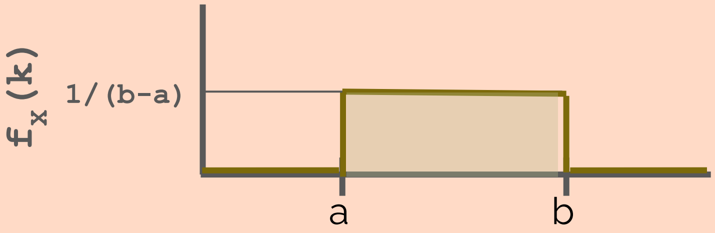
discrete vs. continuous

	Discrete	Continuous
PMF/PDF	$p_X(x) = P(X = x)$	$f_X(x) \neq P(X = x) = 0$
CDF	$F_X(x) = \sum_{t \leq x} p_X(t)$	$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$
Normalization	$\sum_x p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$
Expectation	$\mathbb{E}[g(X)] = \sum_x g(x)\, p_X(x)$	$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$

Zoo of continuous RVs

Uniform RV (continuous version)

X~Unif(a, b) randomly takes on any real number between a and b



$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{a+b}{2}$$

$$\mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential RV

X~Exp(λ) tells how much time till a certain event happens
(λ *is the rate of time*)

think of this as the "continuous version" of the geometric distribution!

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$\mathrm{Var}(X) = \frac{1}{\lambda^2}$$

$$F_X(x) = 1 - e^{-\lambda x}$$

don't confuse this with the Poisson distribution just bc it's related with time, they're very different!
(Poisson is *number* of events in a certain time frame)

$F_X(x)$ = P(X <= x) *this is the integral of $f_X(x)$*

Question 1: “Continuous R.V example”

Suppose that X is a random variable with pdf

$$f_X(x) = \begin{cases} 2C(2x - x^2) & 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

where C is an appropriately chosen constant.

a) What must the constant C be for this to be a valid pdf?

b) Using this C, what is $\mathbb{P}(X > 1)$?

Question 3: “Create the distribution”

Suppose X is a continuous random variable that is uniform on $[0, 1]$ and uniform on $[1, 2]$, but

$$\mathbb{P}(1 \leq X \leq 2) = 2 \cdot \mathbb{P}(0 \leq X < 1).$$

Outside of $[0, 2]$ the density is 0. What is the PDF and CDF of X?

Question 6: “Throwing a dart”

Consider the closed unit circle of radius r, i.e., $S = \{(x, y) : x^2 + y^2 \leq r^2\}$. Suppose we throw a dart onto this circle and are guaranteed to hit it, but the dart is equally likely to land anywhere in S. Concretely this means that the probability that the dart lands in any particular area of size A, is equal to $\frac{\text{A}}{\text{Area of whole circle}}$.
Let X be the distance the dart lands from the center. What is the CDF and pdf of X? What is $\mathbb{E}[X]$ and $Var(X)$?

Midterm Q&A

Any questions for the midterm?

Thank you!