CSE 312: Foundations of Computing II Winter 2024

Problem Set 1

Due: Wednesday, January 10, by 11:59pm

Instructions

Solutions format. Every step in your solution should be explained carefully. The logical reasoning behind your solution should be sound and evident from your write-up.

For example, if you are asked to compute the number of ways to permute the set $\{1, 2, 3, 4\}$ that start with 1 or 2, it is not enough to provide the answer 12. A complete approach would explain that (1) we can count separately the permutations starting with 1 and those starting with 2, and that (2) the two sets are disjoint, and hence the overall number is the sum of the numbers of permutations of each type. Then, (3) explain that there are $3!$ permutations of each type. Finally, (4) say that the overall number totals to $2 \cdot 3! = 12$.

A higher number of mathematical symbols in your solution will not make your solution more precise or "better" – what *is* important is that the logical flow is complete and can be followed by the graders. Relying exclusively on mathematical symbols in fact often make the solution less readable. Avoid expressions such as "it easy to see" and "clearly" – just explain these steps.

Also, you may find the following short note (by Francis E. Su at Harvey Mudd) helpful.

Unless a problem states otherwise, you can leave your answer in terms of factorials, combinations, etc., for instance 26^7 or $26!/7!$ or $26 \cdot \binom{26}{7}$ are all good forms for final answers.

Solutions submission. You must submit your solution via Gradescope. In particular:

- Submit a *single* PDF file containing the solution to all **written** tasks in the homework on Gradescope to **"PSet 1 [Written]"**. Each numbered task should be solved on its own page (or pages). Follow the prompt on Gradescope to link tasks to your pages. Submit your coding assignment on Gradescope to **"Pset 1 [Coding]"**.
- Do not write your name on the individual pages – Gradescope will handle that.
- We encourage you to typeset your solution. The course webpage provides links to resources to help you doing so using LaTeX. We have also provided a template file to start from. If you do use another tool (e.g., Microsoft Word), we request that you use a proper equation editor to display math (MS Word has one). For example, you should be able to write $\sum_{i=1}^{n} x^i$ instead of $\mathrm{x\hat{}1} + \mathrm{x\hat{}2} + \ldots + \mathrm{x\hat{}n}$. You can also provide a handwritten solution, as long as it is on a single PDF file that satisfies the above submission format requirements. It is your responsibility to make sure handwritten solutions are readable – we will *not* grade unreadable write-ups.

Late policy. You have a total of **six** late days during the quarter, but can only use up to two late days on any one problem set. Please plan ahead, as we will not be willing to add any additional late days except in absolute, verifiable emergencies. The final problem set will not be accepted late (however, it will be due only on Friday of the last week of class).

Academic Integrity: Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Each student is expected to do their own work on the problem sets in CSE 312 (though several problem sets will be done in pairs). Either way, students may discuss problem sets with each other as well as the course staff, with the following caveats:

- Do not take away any notes or screenshots from your discussions with others.
- After discussing with others, take a 30 minute break before writing up your solutions.
- Cite the names of all your collaborators somewhere on the first page of your homework.
- On a solo homework, write up your solutions entirely on your own. On a homework done in pairs, the pair should be writing up their solution together, but without any consultation with other pairs.

Excessive collaboration (i.e., beyond discussing problem set questions) can result in honor code violations. Questions regarding acceptable collaboration should be directed to the class instructor prior to the collaboration. **It is a violation of the honor code to copy problem set solutions from others, or to copy or derive them from solutions found online or in textbooks, previous instances of this course, or other courses covering the same topics (e.g., STAT 394/5 or probability courses at other schools).** Copying of solutions from students who previously took this or a similar course is also a violation of the honor code. Finally, **be sure that you are able to explain and/or re-derive anything that you submit.** If we have doubts about whether you did the work on your own, we will ask you to come in and explain your solution to us verbally.

Violations of the above or any other issue of academic integrity are taken very seriously, and may be referred to the University Disciplinary Board. Please refer to the Allen School's Academic Misconduct webpage for a detailed description of what is allowable and what is not.

Task 1 – Softball [10 pts]

Fourteen people (6 children and 8 adults) on a softball team show up for a game.

a) (3 points) How many ways are there to choose 4 players to take the field (order of players selected doesn't matter)?

b) (3 points) How many ways are there to assign 4 players to the positions of catcher, pitcher, 1st baseman and shortstop by selecting players from the 14 people who show up?

c) (4 points) How many ways are there to choose 4 players to take the field where at least one of these players must be an adult?

Task 2 – Counting Words [24 pts]

We want to count the number of strings (an ordered sequence of letters) of length 4 from the English alphabet $\{A, B, \cdots, Z\}$ subject to a number of different constraints. Note that we consider the English alphabet here to consist of 6 *vowels* ($\{A, E, I, O, U, Y\}$) and 20 *consonants*.

How many strings are there which ...

a) ... are only made of vowels?

b) ... are only made of consonants?

c) ... have *exactly* one vowel?

d) ... have *exactly* two vowels?

e) ... have at most two vowels, which may only appear in the third and fourth position?

f) ... have at least one vowel?

As described in problem set description, remember to explain your reasoning **for all problems** – do not just give numbers or unjustified calculations.

Task 3 – Arrangements [12 pts]

How many different ways are there to arrange the letters in the following words?
(a) **poisson** (b) **possessionlessness**

Task 4 – Six card hands [10 pts]

How many ways are there to select 6 cards from a standard deck of 52 cards if we require that all 4 suits are represented? Order doesn't matter.

Task 5 – From here to there [20 pts]

In this problems you will consider paths on the integer grid that start at (0,0) in which every step increments one coordinate by 1 and leaves the other unchanged.

a) (4 points) How many such paths are there from (0,0) to (85, 65)?

b) (4 points) How many such paths are there from (0,0) to (85, 65) that go through (10,35)? How many such paths if they must go through (15,40) instead?

c) (6 points) How many such paths are there from (0,0) to (85, 65) that go through (10,35), but do *not* go through (15,40).

d) (6 points) How many such paths from (0,0) to (85, 65) are there that go through neither of (10,35) nor (15,40) (i.e., that do not go through (10,35) **and** do not go through (15,40))?

Task 6 – Binomial Theorem applications [15 pts]

a) (7 points) What is the coefficient of x^4y^6 in the expansion of $(x - 3y^3)^6$?

b) (8 points) Use the binomial theorem to prove that

$$\sum_{i=0}^{500} \binom{500}{500-i} (-3)^i = 2^{500}$$

Task 7 – A gentle introduction to Python [15 pts]

1. **[Coding]** (10 points) Read the Pset1 Coding lesson on Edstem and follow the directions to complete 5 coding exercises. Then submit the 5 required files from each exercise to **PSet1 [Coding]** on Gradescope. You may resubmit as many times on Gradescope before the deadline, and the score that appears on Gradescope for this part is final.

2. **[Written]** (5 points) Read the Edstem lesson on Python's numpy library, after completing the previous part. You do **not** need to complete any coding exercises or submit anything to Gradescope for this part. The exercise that is there is entirely OPTIONAL, and intended only for practice if you need it. Afterwards, write down what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort rather than correctness, and it's recommended that your answer be no longer than 5 sentences.