CSE 312

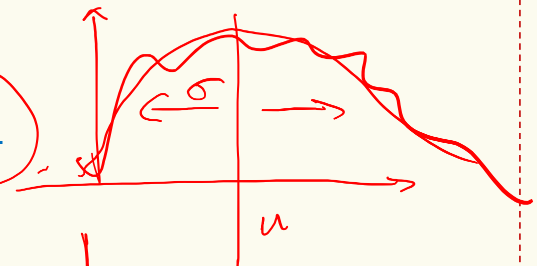
# Foundations of Computing II

## Lecture 16: CLT & Polling

# Review The Normal Distribution


**Definition.** A **Gaussian (or normal) random variable** with parameters $\mu \in \mathbb{R}$ and $\sigma \geq 0$ has density

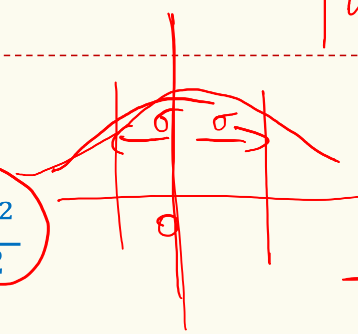
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

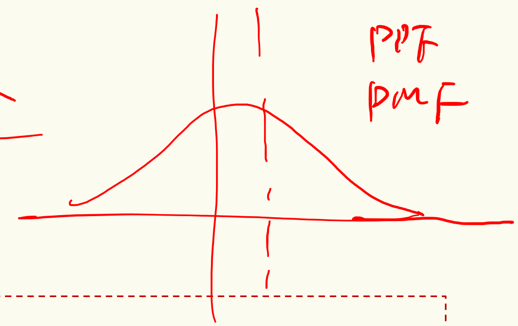



Carl Friedrich Gauss

**Standard (unit) normal** = $\mathcal{N}(0,1)$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$



PDF
PMF

**CDF.** $\Phi(y) = P(Y \leq y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-y^2/2} \mathrm{d}x$

Note: $\Phi(z)$ given via tables

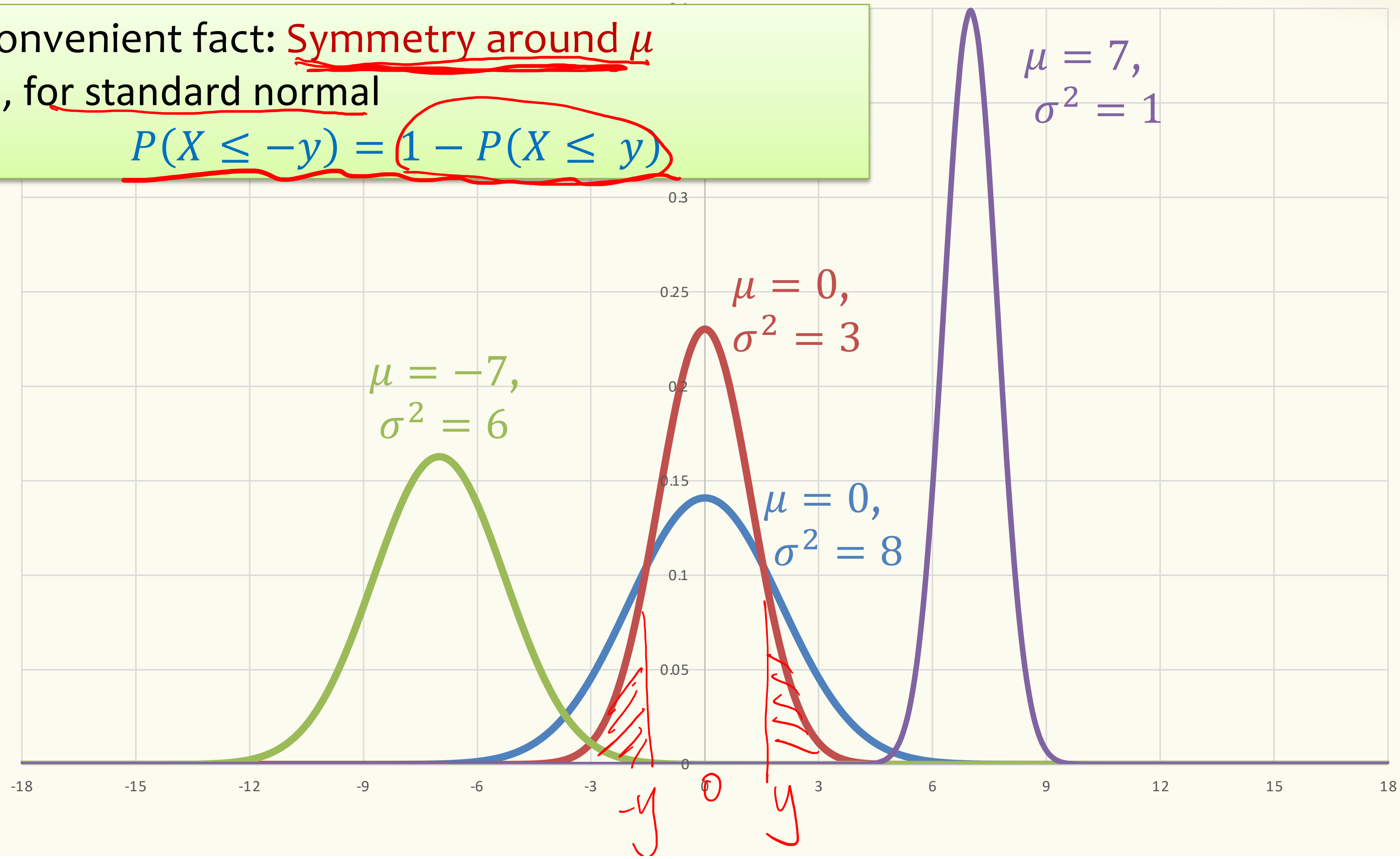
# Review The Normal Distribution

Aka a "Bell Curve" (imprecise name)

A convenient fact: Symmetry around $\mu$

E.g., for standard normal

$$P(X \le -y) = 1 - P(X \le y)$$



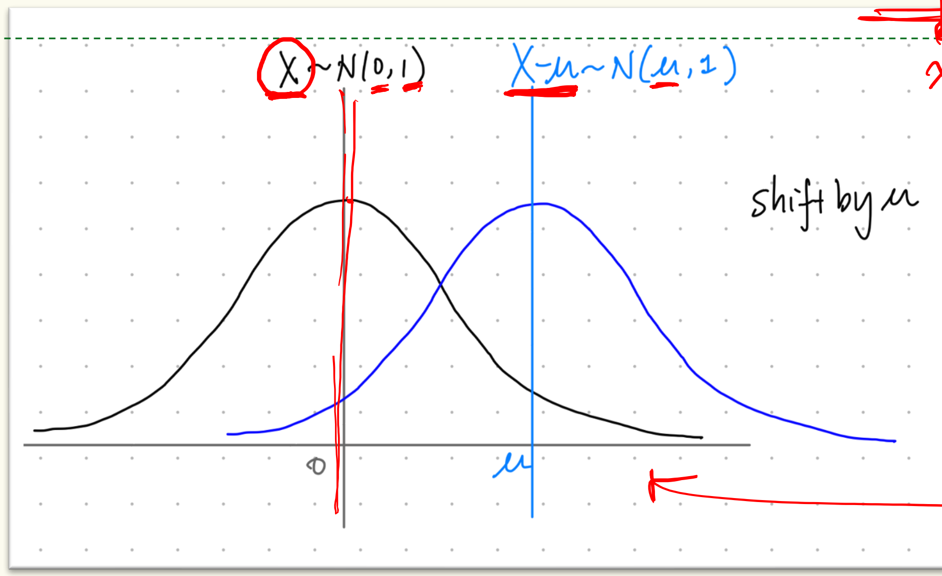
## Review Closure of normal distribution – Under Shifting and Scaling

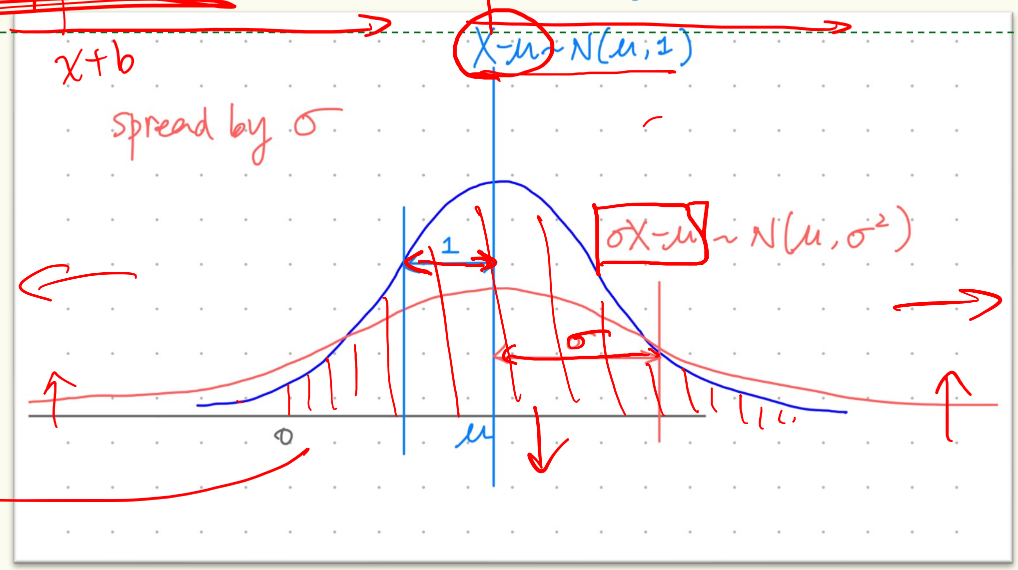
**Fact.** If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Y = aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

**Special Case.** From standard normal $X \sim \mathcal{N}(0,1)$ to general $\sigma X + \mu \sim \mathcal{N}(\mu, \sigma^2)$

**Special Case.** From general normal $X \sim \mathcal{N}(\mu, \sigma^2)$ to standard $\frac{(X-\mu)}{\sigma} \sim \mathcal{N}(0,1)$

x    x+b



## Review How Many Standard Deviations Away?

Let $X \sim \mathcal{N}(\mu, \sigma^2)$.

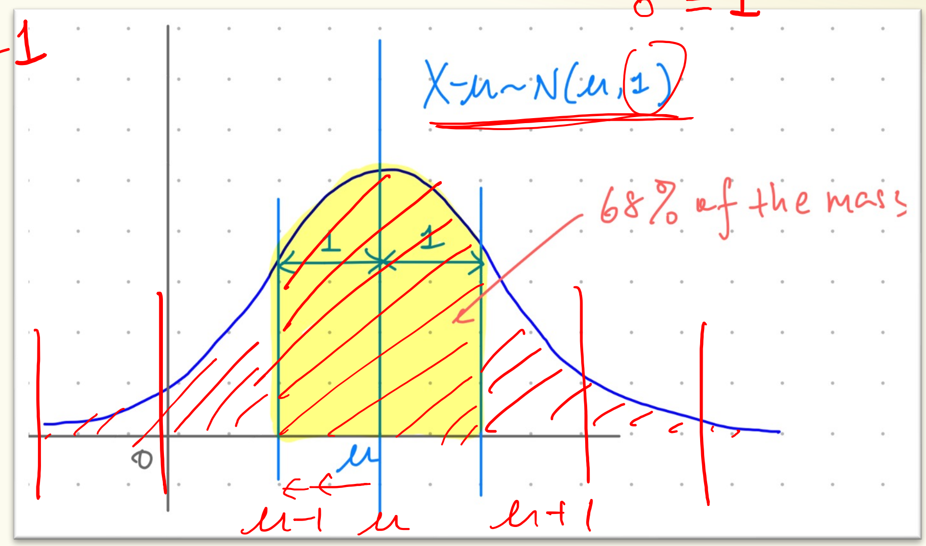
$$P(|X - \mu| < k\sigma) = P\left(\frac{|X - \mu|}{\sigma} < k\right)$$

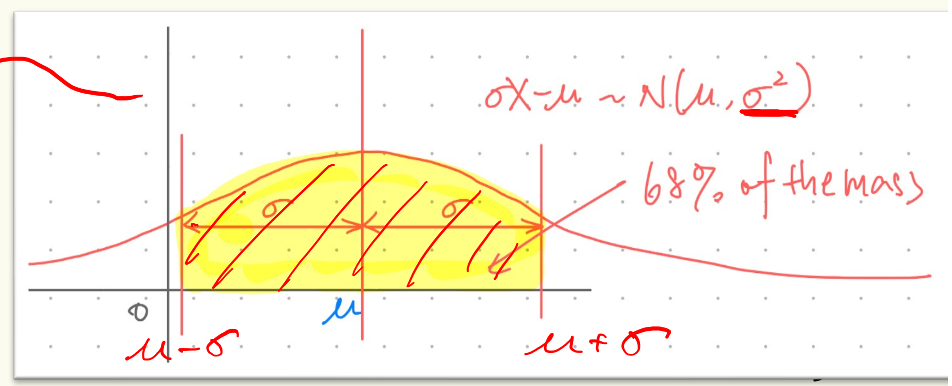
$$= P\left(-k < \frac{X - \mu}{\sigma} < k\right) \sim N(0,1)$$

$$= \Phi(k) - \Phi(-k)$$

e.g. $k = 1$: 68%
$k = 2$: 95%
$k = 3$: 99%

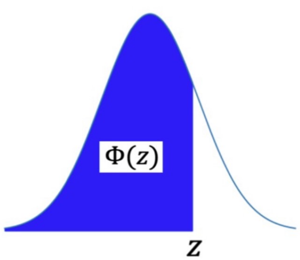

$k=1$ $\sigma = 1$



## Review

# Table of $\Phi(z)$ CDF of Standard Normal Distribution

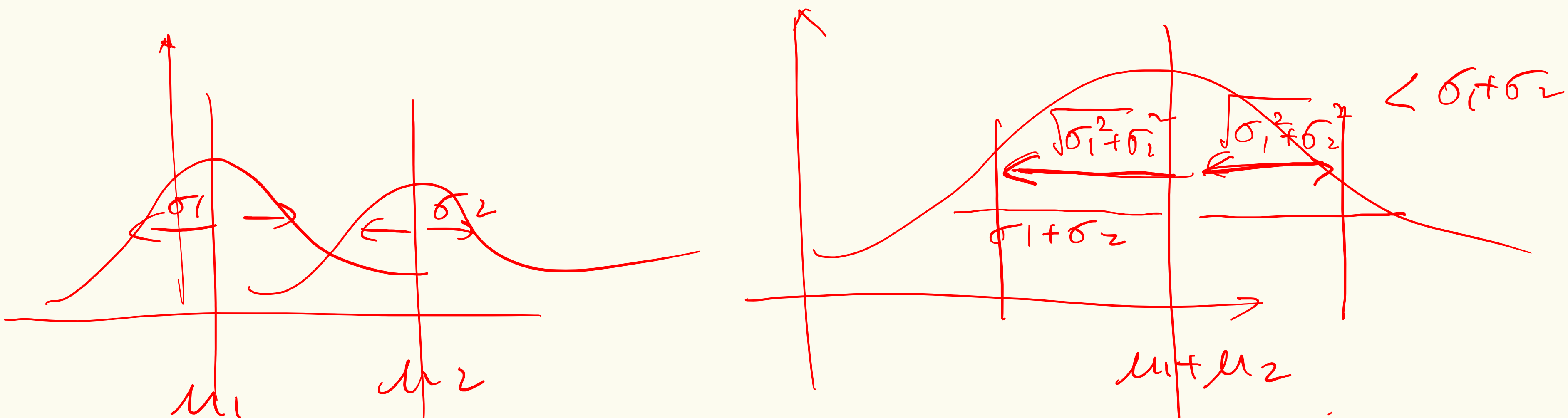



$\Phi$ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0,1)$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.5279 | 0.53188 | 0.53586 |
| 0.1 | 0.53983 | 0.5438 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 |
| 0.2 | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62172 | 0.62552 | 0.6293 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| 0.4 | 0.65542 | 0.6591 | 0.66276 | 0.6664 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.7054 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.7224 |
| 0.6 | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.7549 |
| 0.7 | 0.75804 | 0.76115 | 0.76424 | 0.7673 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.7823 | 0.78524 |
| 0.8 | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| 0.9 | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| 1.1 | 0.86433 | 0.8665 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.879 | 0.881 | 0.88298 |
| 1.2 | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| 1.3 | 0.9032 | 0.9049 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.9222 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.9452 | 0.9463 | 0.94738 | 0.94845 | 0.9495 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.9608 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.9732 | 0.97381 | 0.97441 | 0.975 | 0.97558 | 0.97615 | 0.9767 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.9803 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.983 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.985 | 0.98537 | 0.98574 |
| 2.2 | 0.9861 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.9884 | 0.9887 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | 0.9901 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| 2.4 | 0.9918 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.9943 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.9952 |
| 2.6 | 0.99534 | 0.99547 | 0.9956 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| 2.7 | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.9972 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.9976 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |
| 3.0 | 0.99865 | 0.99869 | 0.99874 | 0.99878 | 0.99882 | 0.99886 | 0.99889 | 0.99893 | 0.99896 | 0.999 |

# Sum of independent normal is still normal

**Fact.** If $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$, $\mathrm{Y} \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$ (both independent normal RV) then $\mathrm{a}X + bY + c \sim \mathcal{N}(a\mu_X + b\mu_Y + c, a^2\sigma_X^2 + b^2\sigma_Y^2)$




$\sigma^2 = \sigma_1^2 + \sigma_2^2$  $\sigma = \sqrt{\sigma_1^2+\sigma_2^2}$

# Agenda

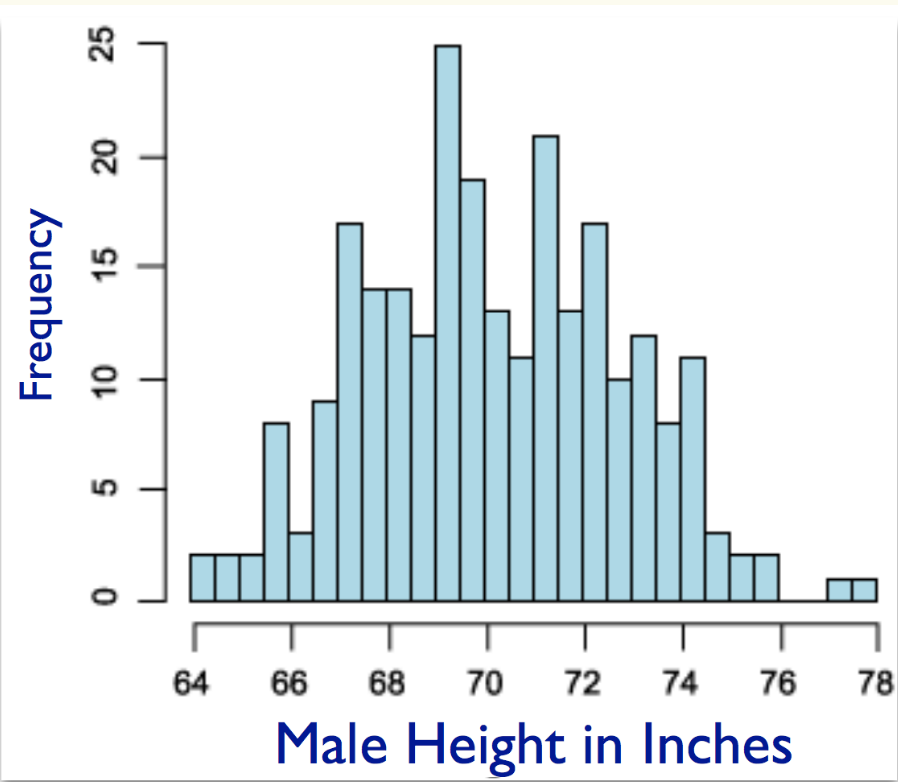
- Central Limit Theorem (CLT)
- Polling

$X, \; Y = -X$

$X, \; Y, \; Z = -X - Y$

## Gaussian in Nature

Empirical distribution of collected data often resembles a Gaussian …




e.g. Height distribution resembles Gaussian.

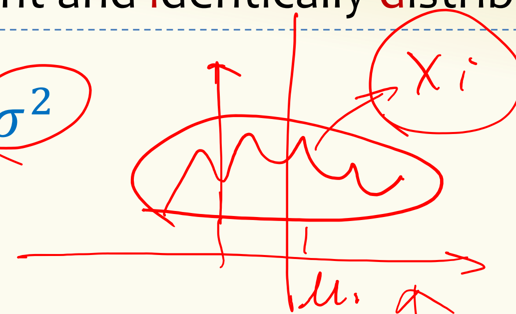
R.A.Fisher (1918) observed that the height is likely the outcome of the sum of many independent random parameters, i.e., can written as

$$X = X_1 + \cdots + X_n$$

# Sum of Independent RVs

i.i.d. = independent and identically distributed

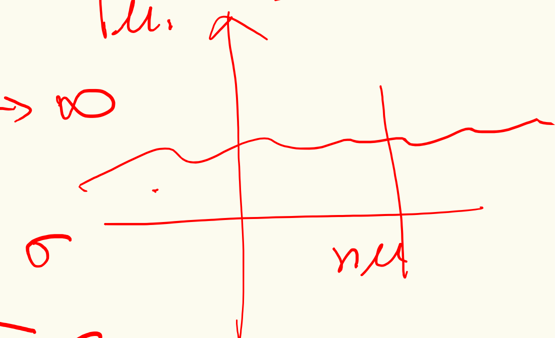
$X_1, \dots, X_n$ i.i.d. with expectation $\mu$ and variance $\sigma^2$




Define

$$S_n = X_1 + \cdots + X_n$$

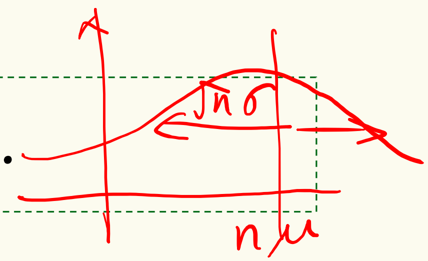
$n \to \infty$




$$\mathbb{E}[S_n] = \mathbb{E}[X_1] + \cdots + \mathbb{E}[X_n] = n\mu$$

$$\mathrm{Var}(S_n) = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_n) = n\sigma^2 \to \sqrt{n}\cdot\sigma$$

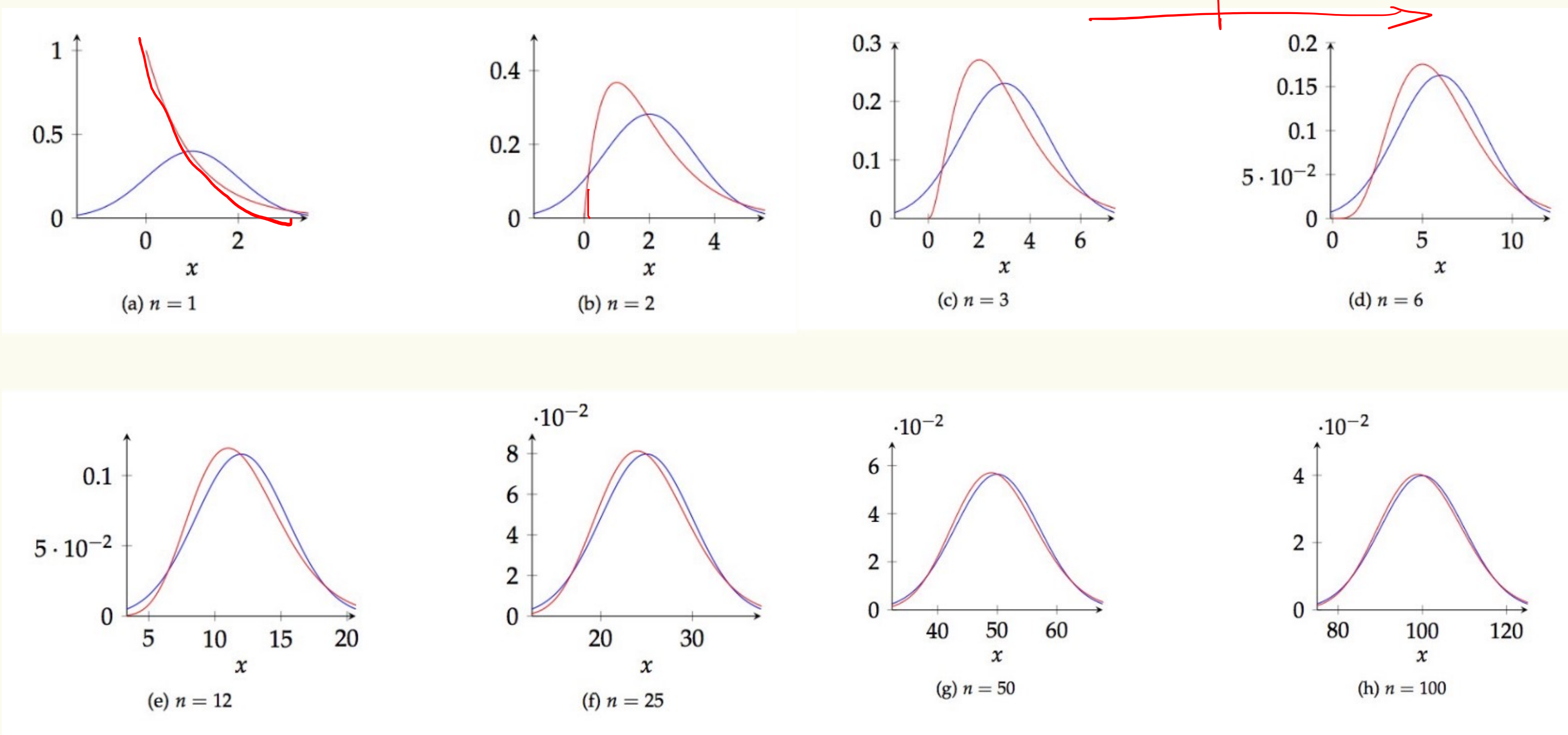
**Empirical observation:** $S_n$ looks like a normal RV as $n$ grows.



# Example: Sum of $n$ i.i.d. $\mathrm{Exp}(1)$ random variables

$n = 1$

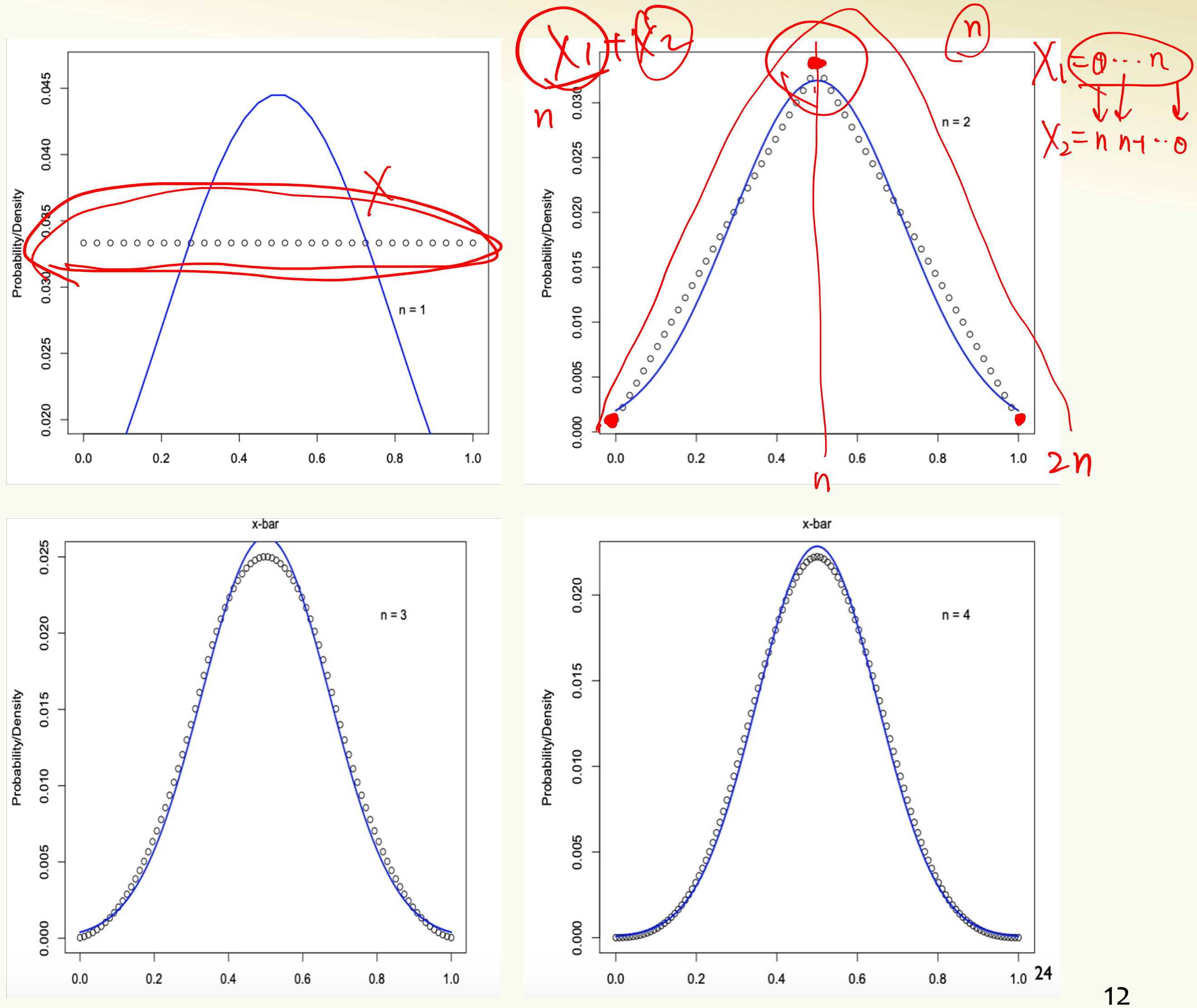
$\mathrm{Exp}(1) + \mathrm{Exp}(1)$




(a) $n = 1$ (b) $n = 2$ (c) $n = 3$ (d) $n = 6$

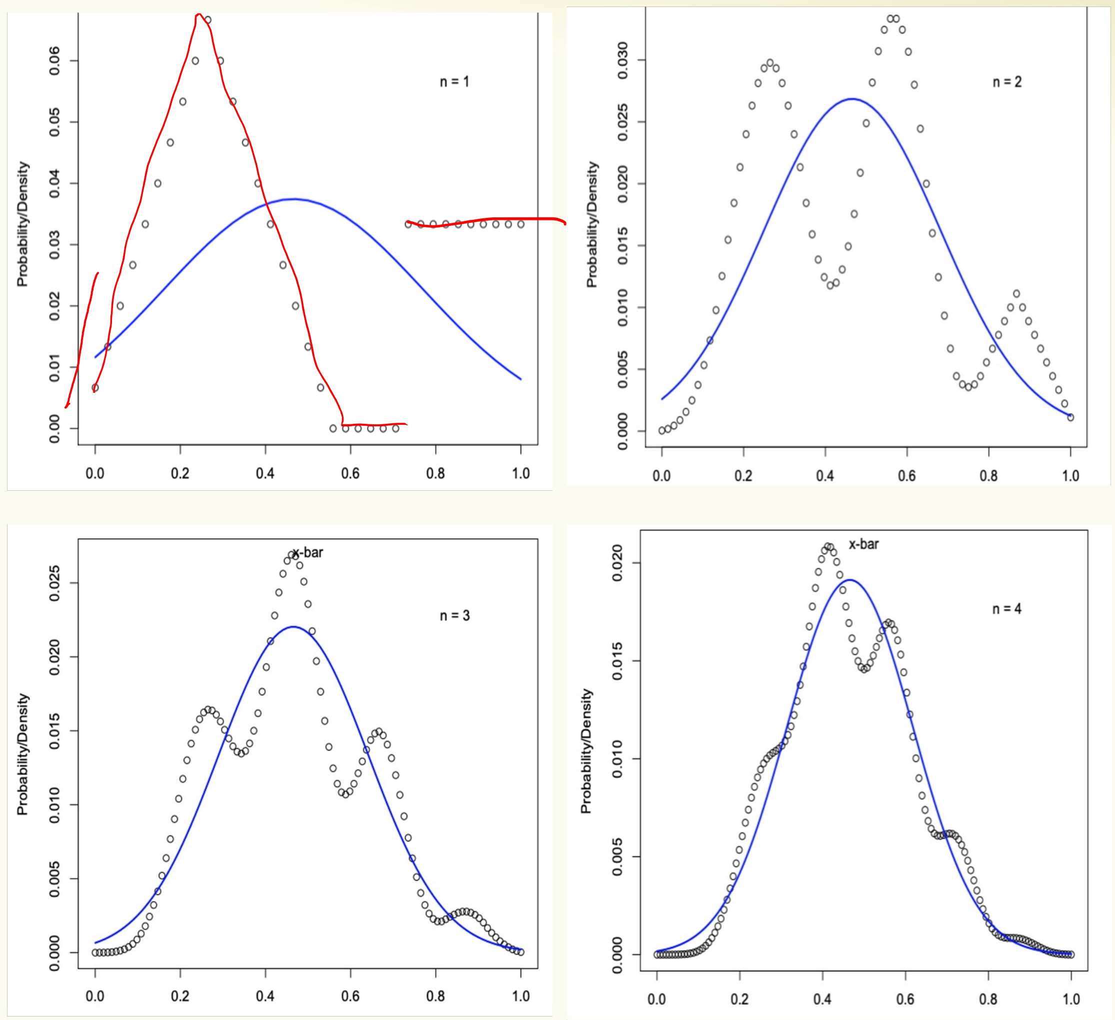
(e) $n = 12$ (f) $n = 25$ (g) $n = 50$ (h) $n = 100$

# CLT (Idea)



Probability/Density
n = 1
n = 2
n = 3
n = 4
x-bar
X
$X_1 + X_2$
n
2n
$X_1 = 0 \cdots n$
$X_2 = n \; n-1 \cdots 0$
24

# CLT (Idea)


n = 1
n = 2
n = 3
n = 4
x-bar
x-bar
Probability/Density
Probability/Density
Probability/Density
Probability/Density
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.000
0.005
0.010
0.015
0.020
0.025
0.000
0.005
0.010
0.015
0.020
0.0
0.2
0.4
0.6
0.8
1.0

# Central Limit Theorem

$X_1, \ldots, X_n$ i.i.d., each with expectation $\mu$ and variance $\sigma^2$

$\rightarrow N(n\mu, n\sigma^2)$

Define $\boxed{S_n = X_1 + \cdots + X_n}$ and

$$\boxed{Y_n = \frac{S_n - n\mu}{\sigma\sqrt{n}}} \rightarrow N(0,1)$$

$$\mathbb{E}[Y_n] = \frac{1}{\sigma\sqrt{n}}(\mathbb{E}[S_n] - n\mu) = \frac{1}{\sigma\sqrt{n}}(n\mu - n\mu) = 0$$

$$\mathrm{Var}(Y_n) = \frac{1}{\sigma^2 n}\big(\mathrm{Var}(S_n - n\mu)\big) = \frac{\mathrm{Var}(S_n)}{\sigma^2 n} = \frac{\sigma^2 n}{\sigma^2 n} = 1$$

## Central Limit Theorem

$X_1, \dots, X_n$ i.i.d., each with expectation $\mu$ and variance $\sigma^2$

Define $S_n = X_1 + \cdots + X_n$ and

$$Z_n = \frac{S_n}{n}$$

$$\mathbb{E}[Z_n] = \frac{n\mu}{n} = \mu$$

$$\mathrm{Var}(Z_n) = \frac{Var(S_n)}{n^2} = \frac{\sigma^2 n}{n^2} = \boxed{\frac{\sigma^2}{n}}$$

## Central Limit Theorem

$\mathcal{N}(0,1) \leftarrow$ $Y_n = \dfrac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$

**Theorem. (Central Limit Theorem)** The CDF of $Y_n$ converges to the CDF of the standard normal $\mathcal{N}(0,1)$, i.e.,

$$\boxed{\lim_{n\to\infty}} P(Y_n \le y) = \boxed{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-x^2/2}\mathrm{d}x} = \Phi(y)$$

## Central Limit Theorem

$$Y_n = \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$$

**Theorem. (Central Limit Theorem)** The CDF of $Y_n$ converges to the CDF of the standard normal $\mathcal{N}(0,1)$, i.e.,

$$\lim_{n\to\infty} P(Y_n \le y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} \mathrm{d}x$$

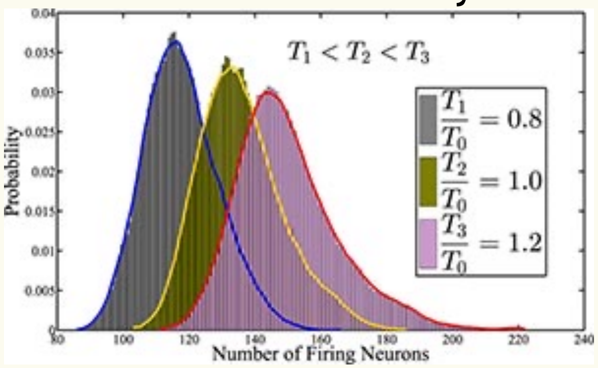
Also stated as:

$X_i \sim N(\mu, \sigma^2)$

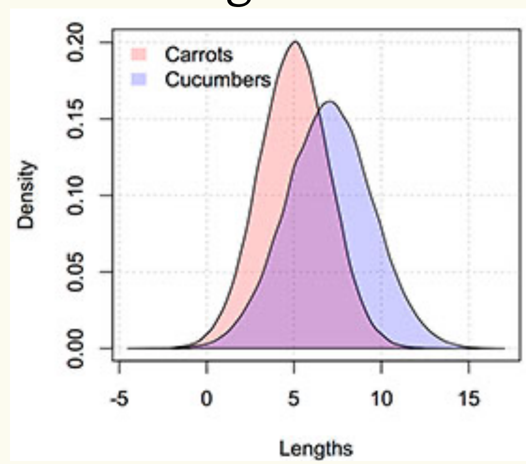
- $\lim_{n\to\infty} Y_n \to \mathcal{N}(0,1)$
- $\lim_{n\to\infty} \underbrace{\frac{1}{n}\sum_{i=1}^{n} X_i}_{Z_n} \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$ for $\mu = \mathbb{E}[X_i]$ and $\sigma^2 = \mathrm{Var}(X_i)$

# CLT → Normal Distribution EVERYWHERE

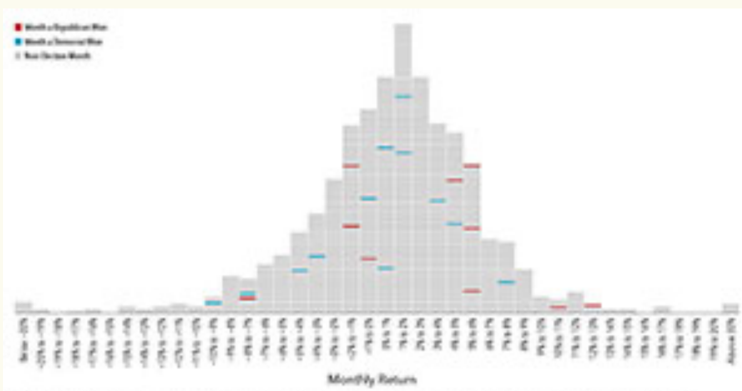
Neuron Activity




Vegetables




S&P 500 Returns after Elections



Examples from:
https://galtonboard.com/probabilityexamplesinlife

## Agenda

- Central Limit Theorem (CLT) Review
- Polling

## Magic Mushrooms

$\bar{p}_1 = 1$ $\bar{p}_n$ 98%

$p = E[\bar{p}]$ $1-p$ $p$ $\bar{p}_1$ 0 1

In Fall 2020, Oregonians voted on whether to legalize the therapeutic use of “magic mushrooms”.

$\bar{p}_1$

Poll to determine the fraction $p$ of the population expected to vote in favor.

- Call up a random sample of $n$ people to ask their opinion $p = E[\bar{p}_1]$
- Report the empirical fraction

empirical

$\bar{p}_n$ = fraction of support in $n$ people

**Questions**

1%

- Is this a good estimate?
- How to choose $n$?

true fraction $p$ .. .. in population

## Polling Accuracy

Often see claims that say

$\bar{p} =$

*"Our poll found 80% support. This poll is accurate to within 5% with 98% probability*"*

$\bar{p} \in [p-5\%, p+5\%]$ $\quad P[\bar{p} \in [p-5\%, p+5\%]] \geq 98\%$

Will unpack what this and how they sample enough people to know this is true.

* When it is 95% this is sometimes written as "19 times out of 20"

# Formalizing Polls

Population size $N$, true fraction of voting in favor $p$, sample size $n$.

**Problem:** We don't know $p$, want to estimate it

## Polling Procedure

for $i = 1, \ldots, n$ :

1. Pick uniformly random person to call (prob: $1/N$)
2. Ask them how they will vote

$$X_i = \begin{cases} 1, & \text{voting in favor} \\ 0, & \text{otherwise} \end{cases}$$

Report our estimate of $p$: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

# Formalizing Polls

What type of r.v. is $X_i$?

Poll: pollev.com/rachel312

| | Type | $\mathbb{E}[X_i]$ | $\mathrm{Var}(X_i)$ |
|---|---|---|---|
| a. | Bernoulli | $p$ | $p(1-p)$ |
| b. | Bernoulli | $p$ | $p^2$ |
| c. | Geometric | $p$ | $\frac{1-p}{p^2}$ |
| d. | Binomial | np | $np(1-p)$ |

Population size $N$, true fraction of voting in favor $p$, sample size $n$.

**Problem:** We don't know $p$

## Polling Procedure

for $i = 1, \ldots, n$ :

1. Pick uniformly random person to call (prob: $1/N$)
2. Ask them how they will vote

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Report our estimate of $p$: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

# Random Variables

What type of r.v. is $X_i$?

| | Type | $\mathbb{E}[X_i]$ | $\mathrm{Var}(X_i)$ |
|---|---|---|---|
| a. | Bernoulli | $p$ | $p(1-p)$ |
| b. | Bernoulli | $p$ | $p^2$ |
| c. | Geometric | $p$ | $\frac{1-p}{p^2}$ |
| d. | Binomial | $np$ | $np(1-p)$ |

What about $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$?

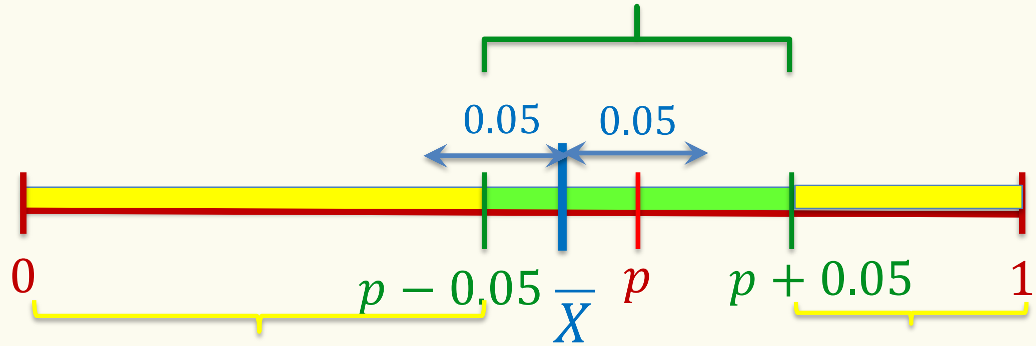
Poll: pollev.com/rachel312

| | $\mathbb{E}[\bar{X}]$ | $\mathrm{Var}(\bar{X})$ |
|---|---|---|
| a. | $np$ | $np(1-p)$ |
| b. | $p$ | $p(1-p)$ |
| c. | $p$ | $p(1-p)/n$ |
| d. | $p/n$ | $p(1-p)/n$ |

## Roadmap: Bounding Error

**Goal:** Find the value of $n$ such that 98% of the time, the estimate $\bar{X}$ is within 5% of the true $p$

Get good estimate if $\bar{X}$ lands in this region




Want $P\left(\left|\bar{X} - p\right| > 0.05\right) \leq 0.02$

## Central Limit Theorem

pollev.com/rachel312

**Poll**: In the limit $\overline{X}$ is…?

a. $\mathcal{N}(0,1)$

b. $\mathcal{N}(p, p(1-p))$

c. $\mathcal{N}(p, p(1-p)/n)$

d. I don't know

With i.i.d random variables $X_1, X_2, \ldots, X_n$ where $\mathbb{E}[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2$

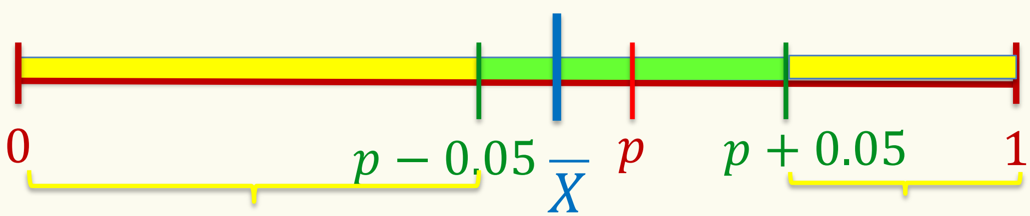
As $n \to \infty$,

$$\frac{X_1 + X_2 + \cdots X_n - n\mu}{\sigma\sqrt{n}} \to \mathcal{N}(0,1)$$

As $n \to \infty$,

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

# Roadmap: Bounding Error



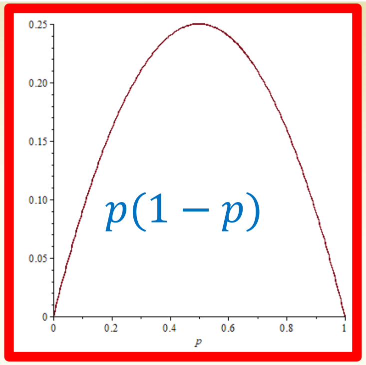

Want $P\left(\left|\overline{X} - p\right| > 0.05\right) \leq 0.02$

## Roadmap: Bounding Error

**Goal:** Find the value of $n$ such that 98% of the time, the estimate $\bar{X}$ is within 5% of the true $p$

1. Define probability of a "bad event" $P(|\bar{X} - p| > 0.05) \leq 0.02$
2. Apply CLT
3. Convert to a standard normal
4. Solve for $n$

# Following the Road Map




1. Want $P\left(\left|\overline{X}-p\right|>0.05\right)\leq 0.02$

2. By CLT $\overline{X}\rightarrow\mathcal{N}(\mu,\sigma^2)$ where $\mu=p$ and $\sigma^2=p(1-p)/n$

3. Define $Z=\frac{\overline{X}-\mu}{\sigma}=\frac{\overline{X}-p}{\sigma}$. Then, by the CLT $Z\rightarrow\mathcal{N}(0,1)$

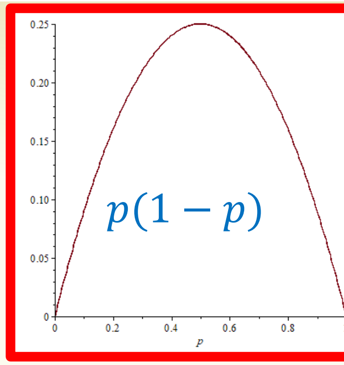
$\boxed{\frac{\mathbf{1}}{\sqrt{\boldsymbol{p}(\mathbf{1}-\boldsymbol{p})}}\text{ is always }\geq\mathbf{2}}$

$$P\left(\left|\overline{X}-p\right|>0.05\right)=P(|Z|\cdot\sigma>0.05)$$

$$=P(|Z|>0.05/\sigma)=P\left(|Z|>0.05\frac{\sqrt{n}}{\sqrt{p(1-p)}}\right)$$

$$\leq P(|Z|>0.1\sqrt{n})$$

# Following the Road Map




1. Want $P\left(\left|\overline{X} - p\right| > 0.05\right) \leq 0.02$

2. By CLT $\overline{X} \to \mathcal{N}(\mu, \sigma^2)$ where $\mu = p$ and $\sigma^2 = p(1-p)/n$

3. Define $Z = \frac{\overline{X}-\mu}{\sigma} = \frac{\overline{X}-p}{\sigma}$. Then, by the CLT $Z \to \mathcal{N}(0,1)$

$\frac{\mathbf{1}}{\sqrt{\boldsymbol{p}(\mathbf{1}-\boldsymbol{p})}}$ is always $\geq \mathbf{2}$

$$P\left(\left|\overline{X} - p\right| > 0.05\right) = P(|Z| \cdot \sigma > 0.05)$$

$$= P(|Z| > 0.05/\sigma) = P\left(|Z| > 0.05 \frac{\sqrt{n}}{\sqrt{p(1-p)}}\right)$$

Want to choose $n$ so that this is at most 0.02

$$\leq P(|Z| > 0.1\sqrt{n})$$

## 4. Solve for $n$

We want $P(|Z| > 0.1\sqrt{n}) \leq 0.02$ *where* $Z \to \mathcal{N}(0,1)$

- If we actually had $Z \sim \mathcal{N}(0,1)$ then enough to show that $P(Z > 0.1\sqrt{n}) \leq 0.01$ since $\mathcal{N}(0,1)$ is symmetric about $0$

- Now $P(Z > z) = 1 - \Phi(z)$ where $\Phi(z)$ is the CDF of the Standard Normal Distribution

- So, want to choose $n$ so that $0.1\sqrt{n} \geq z$ where $\Phi(z) \geq 0.99$

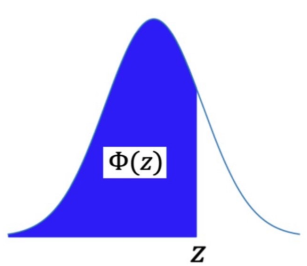
# Table of $\Phi(z)$ CDF of Standard Normal Distribution

Choose $n$ so

$0.1\sqrt{n} \geq z$ where

$\Phi(z) \geq 0.99$

From table $z = 2.33$ works



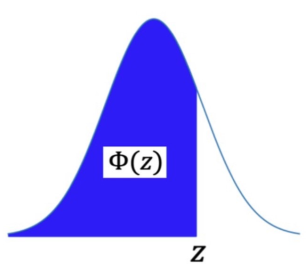

$\Phi$ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0,1)$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.5279 | 0.53188 | 0.53586 |
| 0.1 | 0.53983 | 0.5438 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 |
| 0.2 | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62172 | 0.62552 | 0.6293 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| 0.4 | 0.65542 | 0.6591 | 0.66276 | 0.6664 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.7054 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.7224 |
| 0.6 | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.7549 |
| 0.7 | 0.75804 | 0.76115 | 0.76424 | 0.7673 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.7823 | 0.78524 |
| 0.8 | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| 0.9 | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| 1.1 | 0.86433 | 0.8665 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.879 | 0.881 | 0.88298 |
| 1.2 | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| 1.3 | 0.9032 | 0.9049 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.9222 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.9452 | 0.9463 | 0.94738 | 0.94845 | 0.9495 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.9608 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.9732 | 0.97381 | 0.97441 | 0.975 | 0.97558 | 0.97615 | 0.9767 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.9803 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.983 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.985 | 0.98537 | 0.98574 |
| 2.2 | 0.9861 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.9884 | 0.9887 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | 0.9901 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| 2.4 | 0.9918 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.9943 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.9952 |
| 2.6 | 0.99534 | 0.99547 | 0.9956 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| 2.7 | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.9972 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.9976 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |
| 3.0 | 0.99865 | 0.99869 | 0.99874 | 0.99878 | 0.99882 | 0.99886 | 0.99889 | 0.99893 | 0.99896 | 0.999 |

## 4. Solve for $n$

Choose $n$ so
$0.1\sqrt{n} \geq z$ where
$\Phi(z) \geq 0.99$

From table $z = 2.33$ works




- So we can choose $0.1\sqrt{n} \geq 2.33$ or $\sqrt{n} \geq 23.3$
- Then $n \geq 543 \geq (23.3)^2$ would be good enough … if we had $Z \sim \mathcal{N}(0, 1)$
- We only have $Z \to \mathcal{N}(0, 1)$ so there is some loss due to approximation error.
- Maybe instead consider $z = 3.0$ with $\Phi(z) \geq 0.99865$ and $n \geq 30^2 = 900$ to cover any loss.

## Idealized Polling

So far, we have been discussing "idealized polling". Real life is normally not so nice ☹

Assumed we can sample people uniformly at random, not really possible in practice

- Not everyone responds
- Response rates might differ in different groups
- Will people respond truthfully?

Makes polling in real life much more complex than this idealized model!