Section 1

Review

1) Sum rule. If you can choose from EITHER one of n options, OR one of m options with NO overlap with the previous n, then the number of possible outcomes of the experiment is ______________________.

2) Product rule. In a sequential process with m steps, if there are n_1 choices for the 1st step, n_2 choices for the 2nd step (given the first choice), ..., and n_m choices for the mth step (given the previous choices), then the total number of outcomes is ______________________.

3) Permutations. The number of ways to re-order n elements is _________.

4) Complementary counting. If you can choose from n options, but m of these options are not desirable (and the rest are desirable), then the number of desirable options to choose from is _________.

5) k-permutations. The number of ways to choose a sequence of k distinct elements from a set of n elements is ______.

The rest of these will be covered in class within the next two lectures:

6) Subsets. The number of ways to choose a k-element subset of a set of n elements is ___.

7) Set difference. Is it always true that $|A \backslash B| = |A| - |B|$?

8) Binomial theorem. $\forall x, y \in \mathbb{R}, \forall n \in \mathbb{N}$: $(x+y)^n = \sum_{k=0}^{n} x^k y^{n-k}$

9) Inclusion-exclusion. $|A \cup B| = |A| + |B| - |A \cap B|$.

10) Inclusion-exclusion. $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Task 1 – Sets

a) For each one of the following sets, give its **cardinality**, i.e., indicate how many elements it contains:

- $A = \varnothing$
- $B = \{\varnothing\}$
- $C = \{\{\varnothing\}\}$
- $D = \{\varnothing, \{\varnothing\}\}$

b) Let $S = \{a, b, c\}$ and $T = \{c, d\}$. Compute:

- $S \cup T$
- $S \cap T$
- $S \setminus T$
- $2^{S \setminus T}$
- $S \times T$

Task 2 – Basic Counting

a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is uniquely determined by the first 15 digits). How many credit-card numbers are there?

b) How many positive divisors does $1440 = 2^5 3^2 5$ have?

c) How many ways are there to arrange the CSE 312 staff on a line (12 TAs, one professor) for a group picture?

d) How many ways are there to arrange the CSE 312 staff on a line so that Professor Lin is at one of the ends or exactly in the middle?

Task 3 – Seating

How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

a) . . . all couples are to get adjacent seats?

b) . . . anyone can sit anywhere, except that one couple insists on *not* sitting in adjacent seats?

Task 4 – Lizards and Snakes!

Loudon has three pet lizards (Rango, a gecko named Gordon, and a goanna named Joanna) as well as two small pet snakes (Kaa and Basilisk) but only 4 terrariums to put them in. In how many different ways can he put his 5 pets in these 4 terrariums so that no terrarium has both a snake and a lizard?

Task 5 – Photographs

Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

Task 6 – Weird Card Game

In how many ways can a pack of fifty-two cards (in four suits of thirteen cards each) be dealt to thirteen players, four to each, so that every player has one card from each of the suits?

Task 7 – Birthday Cake

A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert menus are there for the week?

Task 8 – Extended Family Portrait

A group of n families, each with m members, are to be lined up for a photograph. In how many ways can the nm people be arranged if members of a family must stay together?

Task 9 – HBCDEFGA(Not Covered Yet)

How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?