CSE 312: Foundations of Computing II
Quiz Section #7: Zoo of Continuous Random Variables

Review/Mini-Lecture/Main Theorems and Concepts From Lecture

Continuous Random Variable: A r.v. which can take on an uncountably infinite number of values.

Probability Density Function (pdf or density): Let X be a continuous random variable. Then $f_X(x)$ is the density of X. Note that $f_X(x) \neq P(X = x)$, since $P(X = x) = 0$ for all x if X is continuous. However, the probability that X is close to x is proportional to $f_X(x)$: for small δ, $P\left(x - \frac{\delta}{2} < X < x + \frac{\delta}{2}\right) \approx \delta f_X(x)$.

Cumulative Distribution Function (cdf): For a continuous random variable, $P(X \leq x) = F_X(x) = \int_{-\infty}^{x} f_X(t)dt$ and therefore $F_X'(x) = f_X(x)$.

Univariate: Discrete to Continuous:

	Discrete	**Continuous**
PMF/PDF	$p_X(x) = P(X = x)$	$f_X(x) \neq P(X = x)$
CDF	$F_X(x) = \sum_{t \leq x} p_X(t)$	$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$
Normalization	$\sum_x p_X(x) = 1$	$\int_{-\infty}^{\infty} f_X(x)dx = 1$
Expectation	$E[g(X)] = \sum_x g(x)p_X(x)$	$E[g(X)] = \int_{-\infty}^{\infty} g(x)f_X(x)dx$

Zoo of Continuous Random Variables

Uniform: $X \sim Unif(a, b)$ if X has the following probability density function:

$$f_X(x) = \frac{1}{b-a}, \qquad x \in [a, b]$$

$E[X] = \frac{a+b}{2}$ and $Var(X) = \frac{(b-a)^2}{12}$. This represents each real number from $[a, b]$ to be equally likely.

Exponential: $X \sim Exp(\lambda)$ if X has the following probability density function:

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x \geq 0$$

$E[X] = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$. $F_X(x) = 1 - e^{-\lambda x}, x \geq 0$. The exponential random variable is the continuous analog to the geometric random variable: it represents the waiting time to the first success where $\lambda > 0$ is the average number of events per unit time. Note that the exponential measures how much time passes until the first success (any real number, continuous), where the Poisson measures how many events in a unit of time (nonnegative integer, discrete). X is memoryless: for any $s, t \geq 0$, $P(X > s + t \mid X > s) = P(X > t)$. The geometric r.v. also has this property.

Exercises

1. Alex decided he wanted to create a "new" type of distribution that will be famous, but he needs some help. He knows he wants it to be continuous and have uniform density, but he needs help working out some of the details. We'll denote a random variable X having the "Uniform-2" distribution as $X \sim Unif2(a, b, c, d)$, where $a < b < c < d$. We want the density to be non-zero in $[a, b]$ and $[c, d]$, and zero everywhere else. Anywhere the density is non-zero, it must be equal to the same constant.

a) Find the probability density function, $f_X(x)$. Be sure to specify the values it takes on for every point in $(-\infty, \infty)$. (Hint: use a piece-wise definition).

b) Find the cumulative distribution function, $F_X(x)$. Be sure to specify the values it takes on for every point in $(-\infty, \infty)$. (Hint: use a piece-wise definition).

2. Suppose $X \sim Unif(0,1)$ and $Y = e^X$. Find $f_Y(y)$.

3. A single-stranded (1-dimensional) spider web, with length W centimeters, where W>4, is stretched taut between two fence posts. The homeowner (a spider) sits precisely at the midpoint of this web. Suppose that a fly gets caught at a random point on the strand, with each point being equally likely.
a) The spider is lazy, and it is only willing to walk over and eat the fly if the fly lands within 2 centimeters of where the spider sits. What is the probability that the spider eats the fly?

b) Let X be the random variable that represents the spider's distance from the fly's landing point. Calculate the CDF, PDF, expectation, and variance of X.