

CSE 312: Foundations of Computing II  
Supplementary Exercises #2

1. Given 3 spades and 3 hearts, shuffle them. Compute  $P(E)$ , where  $E$  is the event that the suits of the shuffled cards are in alternating order. What is your sample space?
2. Suppose you pick two cards from a well-shuffled Schnapsen deck. What is the probability that they are both queens?
3. At a card party, someone brings out a deck of bridge cards (4 suits with 13 cards in each).  $N$  people each pick 2 cards from the deck. What is the minimum value of  $N$  that guarantees at least 2 people have the same combination of suits?
4. Suppose you deal 13 cards from a well-shuffled bridge deck (4 suits with 13 cards in each). What is the probability that the distribution of suits is 4, 4, 3, 2? (That is, you have 4 cards of one suit, 4 cards of another suit, 3 cards of another suit, and 2 cards of the last suit.)
5. Novice poker players are often confused about whether a flush beats a straight. For draw poker (see quiz section #1 worksheet, exercise #21):
  - (a) Compute the probability of being dealt a flush.
  - (b) Compute the probability of being dealt a straight.
  - (c) Which of these hands should beat the other, given your answers to (a) and (b)?
6. This is another poker exercise. Find the minimum number of cards to be dealt to you from a bridge deck to guarantee that you have some 5 cards among them that form ...
  - (a) one pair? (This occurs when the cards have ranks  $a, a, b, c, d$ , where  $a, b, c$ , and  $d$  are all distinct. The suits do not matter.)
  - (b) two pairs? (This occurs when the cards have ranks  $a, a, b, b, c$ , where  $a, b$ , and  $c$  are all distinct. The suits do not matter.)
  - (c) a full house? (This occurs when the cards have ranks  $a, a, a, b, b$ , where  $a$  and  $b$  are distinct. The suits do not matter.)
  - (d) a straight? (A hand is said to form a straight if the ranks of all 5 cards form an increasing sequence. The suits do not matter. The lowest straight is A, 2, 3, 4, 5 and the highest straight is 10, J, Q, K, A.)
  - (e) a flush? (A hand is said to form a flush if all 5 cards are from the same suit.)
  - (f) a straight flush (5 cards of the same suit that form a straight)?
7. In Schnapsen, suppose that  $\spadesuit J$  is the face-up trump and you are dealt 5 nontrump cards. Let  $E$  be the event that the top 4 cards in the stock are all trumps. Let the sample space be all possible orderings of all the cards in the stock. Compute  $P(E)$ . (Notice that your solution suggests a different and simpler sample space.)

8. Suppose you are taking a multiple-choice test that has  $c$  answer choices for each question. In answering a question on this test, the probability that you know the correct answer is  $p$ . If you don't know the answer, you choose one at random. What is the probability that you knew the correct answer to a question, given that you answered it correctly?
9. An urn contains 3 black balls and 4 white balls.
  - (a) Suppose 3 balls are drawn from the urn without replacement. What is the probability that all 3 are white? Try computing this in the sample space where the order of the 3 draws does not matter, and then in the sample space where the order does matter.
  - (b) Suppose 3 balls are drawn from the urn with replacement. What is the probability that all 3 are white? Describe the sample space precisely.
10. At a dinner party, the  $n$  people present are to be seated uniformly spaced around a circular table. Suppose there is a nametag at each place at the table and suppose that nobody sits down at the correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.
11.
  - (a) Two parents only have 3 bedrooms for their 13 children. If each child is assigned to a bedroom, one of the bedrooms must have at least  $c$  children. What is the maximum value of  $c$  that makes this statement true? Prove it.
  - (b) (Strong Pigeonhole Principle) More generally, what can you say about  $n$  children in  $k$  bedrooms? Find a general formula for the maximum value of  $c$  that guarantees one of the bedrooms must have at least  $c$  children.
12. Twenty politicians are having a tea party, 6 Democrats and 14 Republicans.
  - (a) If they only give tea to 10 of the 20 people, what is the probability that they only give tea to Republicans?
  - (b) If they only give tea to 10 of the 20 people, what is the probability that they give tea to 9 Republicans and 1 Democrat?
13. A couple has 2 children. What is the probability that both are girls, given that the older one is a girl?
14. (Challenge problem)  $n$  people at a reception give their hats to a hat-check person. When they leave, the hat-check person gives each of them a hat chosen at random. What is the probability that no one gets their own hat back?