CSE 312 Practice Midterm

Q1. A small scale distributed file system consists of 12 servers. We number them from 1 to 12 respectively. According to historical data, each server crashes independently with probability $\frac{1}{6}$ every hour. Answer the following for one particular hour:

a) What is the probability both the 3rd server and the 6th server crash?

b) The entire system will continue to function as long as more than (but not including) $\frac{3}{4}$ of the servers are working. What is the probability that the entire system will fail? (No need to simplify your answer.)

Q2. There's a popular social network where two people are considered to be connected if both of them add each other as "Friends". The structure of this social network is determined by which pairs of people are "Friends". If there are n active users on the social network, how many possibilities are there for the structure of this social network?

Q3. I have a pile of 6 (identical-looking) coins:

- 3 of them are fair coins: P(heads) = ½
- 2 of them are biased such that P(heads) = 1
- 1 of them is biased such that P(heads) = ¾

Suppose I draw a coin randomly, with each coin being equally likely. Then I flip the coin 6 times, and it comes up heads 4 times. What is the probability that the coin I flipped was one of the fair coins? No need to simplify your answer.

Q4. There is a train with 3 carriages which are all initially empty. It arrives at a train station where 20 people are waiting to get on. Each person chooses to get on each of the carriages with equal probability. How many ways are there for people to get on the train so that none of the carriages are empty? You may leave your answer as a simplified expression.

Q5. Suppose we throw n balls into n bins with the probability of a ball landing in each of the n bins being equal. What is the expected number of empty bins?

Q6. A negative binomial r.v. is denoted $X \sim NegBin(r, p)$. X is the number of independent trials up to and including the rth success, where $P(success) = p$ for any single trial.

a) X can be written as a sum of iid (independent and identically distributed) r.v.'s, $X_1 + X_2 + \dots + X_r$, where each X_i follows the ________________ distribution with parameter ____.

b) Find the pmf $p(x)$ for X. For which values of x is this pmf defined?

c) Find $E[X]$.

d) Find $Var(X)$.