Continuous Random Variables
A continuous r.v. takes values from an uncountable set, \( \mathbb{R} \), the real numbers.
Ex: weight of a randomly chosen person; waiting time until arrival of next packet.

Defn: \( f: \mathbb{R} \to \mathbb{R} \) is a probability density function (pdf) (density),
iff \( \forall x \in \mathbb{R}, f(x) \geq 0 \) and \( \int_{-\infty}^{+\infty} f(x) \, dx = 1 \).
(Normalized to 1, like a PMF.)

Defn: The cumulative distribution function (CDF)
\( F: \mathbb{R} \to [0,1] \) associated with pdf \( f \) is
\[
F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x) \, dx
\]

\[
\begin{array}{c}
\text{Diagram:}
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\]

\( F(a) \)

\( P(a < X \leq b) = F(b) - F(a) = \int_{a}^{b} f(x) \, dx = P(X \leq b) - P(X \leq a) \)

(2) \( f(x) = \frac{d}{dx} F(x) \).

(3) Densities are not probabilities; they may be \( > 1 \).

(4) \( P(X = a) = F(a) - F(a) = 0 \).