<u>Ex:</u> One roll of a fair 6-sided die,
$X \sim \text{Unif}(1,6)$,
$E[X] = \frac{1}{2}(1+6) = \frac{7}{2}$
$\text{Var}(X) = \frac{1}{12}(6-1)(6-1+2) = \frac{35}{12}$.

<u>Defn:</u> A <u>Bernoulli random variable</u> $X \sim \text{Ber}(p)$ is a random indicator variable with
$P(X=1) = p$ and $P(X=0) = 1-p$.
$E[X] = 1 \cdot P(X=1) + 0 \cdot P(X=0) = p$
$E[X^2] = 1^2 \cdot P(X=1) + 0^2 \cdot P(X=0) = p$
$\text{Var}(X) = E[X^2] - (E[X])^2 = p - p^2 = p(1-p)$

<u>Ex:</u> One flip of a coin that has $P(\text{heads}) = p$.

<u>Defn:</u> A <u>binomial random variable</u> $X \sim \text{Bin}(n,p)$ is the sum of n independent Bernoulli random variables $X_i \sim \text{Ber}(p)$, for $1 \leq i \leq n$.
$P(X=j) = \cancel{\binom{n}{i} p^i (1-p)^{n-i}} \binom{n}{j} p^j (1-p)^{n-j}$

T H T T H H T T H T n slots

i heads

$E[X]$:
Let $X_1, X_2, \ldots X_n \sim \text{Ber}(p)$ and independent, and $X = \sum_{i=1}^{n} X_i$.
$E[X] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i]$ (lin. of E)
$= \sum_{i=1}^{n} p = np$

$$Var(X) = Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i) \quad (X_i\text{'s are ind.})$$
$$= \sum_{i=1}^{n} p(1-p) = np(1-p).$$

Ex: # of heads in n ind flips of a coin w/ $P(\text{heads}) = p$.