Ex: n chips manufactured, d of them defective. k chips selected randomly from the n for testing. What is

P(k selected chips contain some defective chip)?

Let Ω be all the ways of choosing a set of k chips from the n chips.

Let E be all ways of choosing a set of k chip with none of them defective. (Complement).

$$P(\text{none of } k \text{ defective}) = P(E) = \frac{\binom{n-d}{k}}{\binom{n}{k}}$$

$P(\geq 1 \text{ of } k \text{ defective})$

$$= P(\bar{E}) = 1 - P(E) = 1 - \frac{\binom{n-d}{k}}{\binom{n}{k}}.$$

Conditional Probability of E given F, written $P(E|F)$, where $F \neq \emptyset$, is the probability that E occurs, given that F was observed. Sample space reduced to F, event is reduced to $E \cap F$.

With equally likely outcomes,

$$P(E|F) = \frac{|E \cap F|}{|F|}$$

$$= \frac{|E \cap F| / |\Omega|}{|F| / |\Omega|} = \frac{P(E \cap F)}{P(F)}$$

Ex: Roll a fair die. What is $P(5 \mid \text{odd})$

From counting: $P(E|F) = |E \cap F| / |F| = |5 \cap \text{odd}| / |\text{odd}| =$

From prob: $P(E|F) = P(E \cap F)/P(F) = P(5)/P(\text{odd}) = \frac{1/6}{1/2} = \frac{1}{3}$

Ex: Let Ω be all ways of dealing 2 Schnapsen hands,
Y = you are dealt 0 trumps, and
Θ = opponent is dealt 0 trumps.

$$P(\Theta|Y) = \frac{|\Theta \cap Y|}{|Y|} = \frac{\cancel{\binom{15}{5}}\binom{10}{5}}{\cancel{\binom{15}{5}}\binom{14}{5}} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{14\cdot 13\cdot 12\cdot 11\cdot 10} \approx 0.126$$

Compare to $P(\Theta) \approx 0.258$

Ex: $P(\text{opp dealt} \geq 1 \text{ trump} \mid Y) = P(\bar{\Theta}|Y) = 1 - P(\Theta|Y) \approx 0.87$

$$P(\Theta|\bar{Y}) \neq 1 - P(\Theta|Y)$$