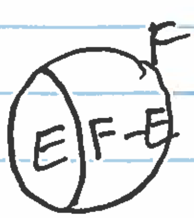
Probability

1. A sample space is a set Ω of possible "outcomes" of an experiment.
   Ex: Coin flip. $\Omega = \{H, T\}$
   2 coin flips. $\Omega = \{(H,H), (H,T), (T,H), (T,T)\}$
   Die roll: $\Omega = \{1,2,3,4,5,6\}$
   # emails I get in 1 day: $\Omega = \{x \in \mathbb{Z} \mid x \geq 0\} = \mathbb{N}$
   5-card Schnapsen hands, assuming ♣J is face-up
   $|\Omega| = \binom{19}{5}$
2. An event is any $E \subseteq \Omega$.
   Ex. ≥ 1 head in 2 coin flips. $E = \{(H,H), (H,T), (T,H)\}$
   odd roll of a die. $E = \{1,3,5\}$
   5-card Schnapsen hands with no trump. $|E| = \binom{15}{5}$
3. Defn: E and F are mutually exclusive iff
   $E \cap F = \emptyset$.
4. Axioms of probability.
   ~~(1) P(E)~~ There is a function P that assigns a real number $P(E)$ to every event E satisfying:
   (1) $P(E) \geq 0$,
   (2) $P(\Omega) = 1$, and
   (3) If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.
5. Implications of the axioms.

   (a) $P(\overline{E}) = 1 - P(E)$, because
   $1 = P(\Omega) = P(E \cup \overline{E}) = P(E) + P(\overline{E})$

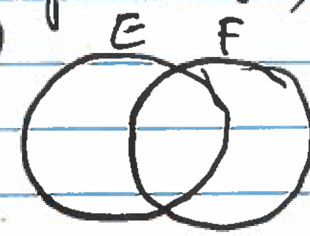


   (b) If $E \subseteq F$, then $P(E) \leq P(F)$, because
   $P(F) = P(E \cup (F-E)) = P(E) + P(F-E) \geq P(E)$ (Ax. 1)

(c) $P(E) \leq 1$ because $E \subseteq \Omega$ and implication (b).

(d) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Inclusion-Exclusion.



F. Equally likely outcomes: Ω is finite and $P(\{a\}) = \frac{1}{|\Omega|}$ for every $a \in \Omega$.

Ex: 1 coin flip of a fair coin: $P(H) = P(T) = \frac{1}{2}$.

1 roll of a fair die: $P(i) = \frac{1}{6}$ for $i \in \Omega$.

Ex: Assume J is face up. Assume any 5-card hand is equally probable.

$$P(\text{no trump card in hand}) = \frac{\binom{15}{5}}{\binom{19}{5}} = \frac{3003}{11628} \approx 0.258$$

In general, $P(E) = \sum_{a \in E} P(a) = \sum_{a \in E} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}$.

Ex: Assume your 5-card hand is dealt before the face-up trump.

$$P(\geq 1 \text{ marriage in your hand}) = \frac{\binom{4}{1}\binom{18}{3} - \binom{4}{2}\binom{16}{1}}{\binom{20}{5}} \approx 0.204$$

Note: change in Ω from previous example.