Complementing.

Ex: Assume QJ is the face-up trump. How many starting Schnapsen hands contain $\geq 1$ trump?

$12\, 240 = (4)\, (4)\, (10)\, (18)$ cards remaining in the deck

$12\, 240 = (1)\, (4)\, (4)\, (10)$ cards I still need to be dealt.

QA 8K D KQJ

8K QA QK QJ

QA 8K QK QJ

8K QA 8Q QJ

8Q QA 8K QJ

Overcounted!

How many contain 0 trumps? (15)

How many contain $\geq 1$? (19) - (15)

$\begin{align*}
\frac{19\cdot 18\cdot 17\cdot 16\cdot 15}{5\cdot 4\cdot 3\cdot 2\cdot 1} - \frac{15\cdot 14\cdot 13\cdot 12\cdot 11}{5\cdot 4\cdot 3\cdot 2\cdot 1} &= 8625
\end{align*}$

Inclusion-Exclusion:

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C|$

$- |A \cap B| - |A \cap C| - |B \cap C|$

$+ |A \cap B \cap C|$
In general: single pairs + triples + quad...

Ex: Correct the overcounting in \( (4)(4) \):

\[
(4)(4)(4) - (4)(4)(3) + (4)(4)(2) - (4)(4)(1)
\]

\[
= 12,240 - 4,080 + 4,800 - 15 = 8,625
\]

Pigeonhole Principle: If you have \( n \) pigeons and \( k \) pigeonholes, where \( k < n \), then some pigeonhole has \( >1 \) pigeon.

Ex:

5x5 chessboard, if flea on each square. When I ring a bell, every flea jumps to an adjacent (horizontally or vertically) square. After\n
that some square has \( \geq 2 \) fleas.

13 white, 12 black.

13 fleas jump onto 12 squares.