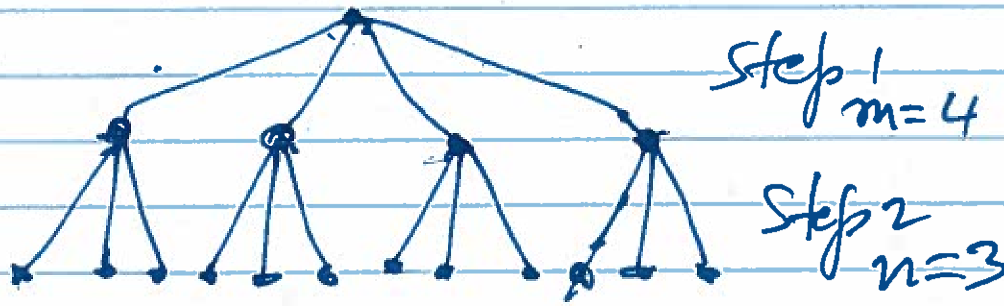
Product Rule

If there are m choices for step 1 and n choices for step 2, there are mn choices for both steps together.



Generalizes to s sequential steps.

Ex: How many n-bit binary strings are there?

$$\underbrace{2\cdot 2\cdot 2\cdots 2}_{n} = 2^n$$

Ex: How many subsets does a set of size n have?
For each element, you have 2 choices, either in the subset or not. 2^n

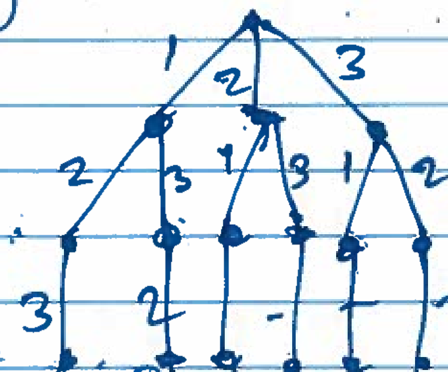
Ex: How many 7-digit phone numbers are there, if the first digit cannot be 0 or 1?
8×10^6

Ex: How many 4-character passwords using lower case letters and digits? $36^4 \approx 1.7\times 10^6$

Ex: How many if no character is repeated?
$36\cdot 35\cdot 34\cdot 33 \approx 1.4\times 10^6$

Permutations: order matters

How many sequences using 1, 2, 3 each exactly once?



In general, the number of permutations of n distinct objects is

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

k-permutations

$$P(n,k) = n(n-1)\cdots(n-k+1)$$

$$= \frac{n!}{(n-k)!}$$

Ex. Back to our password example

$$P(36,4) = \frac{36!}{32!}$$