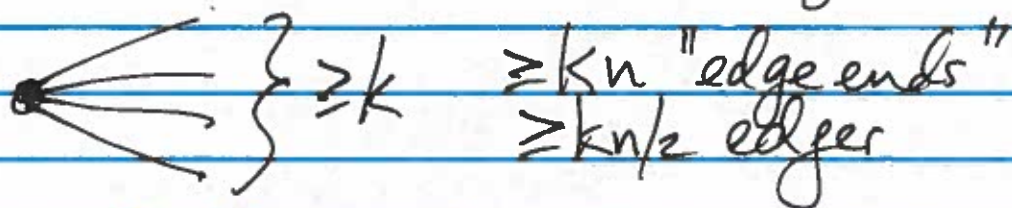


Let  $C$  be a min-cut of  $k$  edges.

Compute the probability that no edge in  $C$  is ever contracted.

A graph with  $n$  vertices and min-cut size  $k$  must have at least  $kn/2$  edges.



Karger's algorithm does  $n-2$  contractions, since each contraction reduces the number of vertices by 1.

Let  $E_i$  be the event that the  $i$ th contraction does not contract an edge in  $C$ , for  $1 \leq i \leq n-2$ .

$$P(\bar{E}_1) \leq \frac{k}{kn/2} = \frac{2}{n}, \text{ so } P(E_1) \geq 1 - \frac{2}{n}$$

$$P(\bar{E}_2 | E_1) \leq \frac{k}{k(n-1)/2} = \frac{2}{n-1}, \text{ so } P(E_2 | E_1) \geq 1 - \frac{2}{n-1}$$

⋮

$$P(\bar{E}_i | E_1 \cap E_2 \cap \dots \cap E_{i-1}) \leq \frac{k}{k(n-i+1)/2} = \frac{2}{n-i+1},$$

$$\text{so } P(E_i | E_1 \cap \dots \cap E_{i-1}) \geq 1 - \frac{2}{n-i+1}$$

$$P(E_1 \cap E_2 \cap \dots \cap E_{n-2})$$

$$= P(E_1)P(E_2 | E_1)P(E_3 | E_1 \cap E_2) \dots P(E_{n-2} | E_1 \cap \dots \cap E_{n-3})$$

=

$$\begin{aligned} \prod_{i=1}^{n-2} \left(1 - \frac{2}{n-i+1}\right) &= \prod_{i=1}^{n-2} \frac{n-i-1}{n-i+1} \\ &= \frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdot \frac{n-4}{n-2} \cdot \frac{n-5}{n-3} \cdots \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \\ &= \frac{2}{n(n-1)} > \frac{2}{n^2} \end{aligned}$$

so the probability of error is  $< 1 - \frac{2}{n^2}$

Run Karger's algorithm  $\frac{tn^2}{2}$  times, where  $t$  is a parameter to be decided, each time using independent choices of the contracted edges. At the end, output the smallest of these  $tn^2/2$  cuts.

The probability that  $C$  is not output in any of these  $tn^2/2$  iterations is

$$\left(1 - \frac{2}{n^2}\right)^{tn^2/2} \leq \left(\frac{1}{e}\right)^t$$

When  $t=14$ , this is  $< 10^{-6}$

This is simpler than the most efficient deterministic algorithm based on network flow.

