#2: Coin 1 : $P(H) = 0.01$ \
Coin 2 : $P(H) = 0.99$ } choose a coin uniformly, and flip twice

$H_1 = P(\text{flip 1 is } H)$

$H_2 = P(\text{flip 2 is } H)$

Are H_1 and H_2 ind?

$P(H_1) = P(H_2) = \frac{1}{2}$

$P(H_2 | H_1) \approx 0.98$

$P(H_2 | H_1) \neq P(H_2)$ so dependent.

#5: $X_1, X_2, \ldots, X_n$ are i.i.d. r.v.'s (independent, identically distributed).

$E[X_i] = \mu, \quad Var(X_i) = \sigma^2$

$M_n = \frac{1}{n} \sum_{i=1}^{n} X_i$

$$E[M_n] = E\left[\frac{1}{n} \sum_{i=1}^{n} X_i\right] = \frac{1}{n} E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{1}{n} \cdot n\mu = \mu$$

$$Var(M_n) = Var\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right)$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} Var(X_i) \qquad \text{because } X_i\text{'s ind}$$

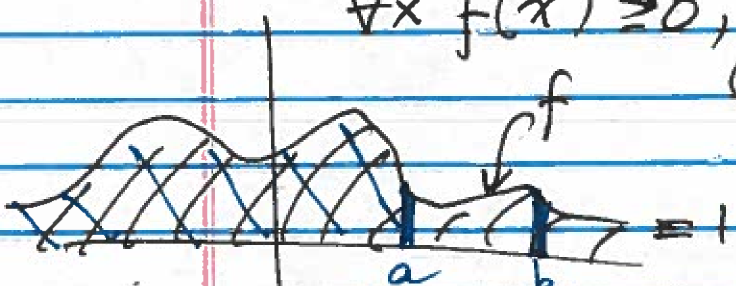
$$= \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2} \cdot n\sigma^2 = \frac{\sigma^2}{n}$$

Continuous Random Variables (slide pack 7)

A continuous r.v. takes values from an uncountable set. (For us, either the real numbers $\mathbb{R}$ or some interval of reals.)

Ex: height of a randomly chosen adult
time until next packet arrives at a server

Defn: $f:\mathbb{R}\to\mathbb{R}$ is a probability density function (pdf) (or density) for a continuous random variable X iff
$\forall x\ f(x)\geq 0$, and $\int_{-\infty}^{+\infty} f(x)\,dx = 1$
(normalized, just like a pmf, where $\sum_x p(x)=1$.)

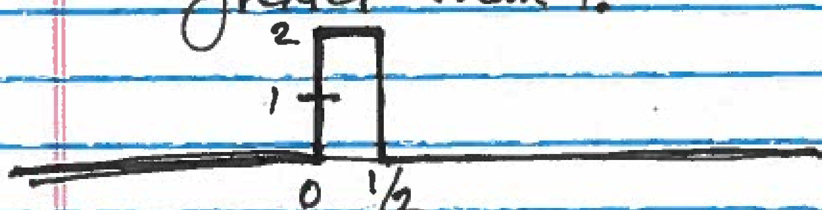


Defn: The cumulative distribution function (cdf) $F:\mathbb{R}\to\mathbb{R}$ associated with pdf f is
$$F(a)=P(X\leq a)=\int_{-\infty}^{a} f(x)\,dx$$

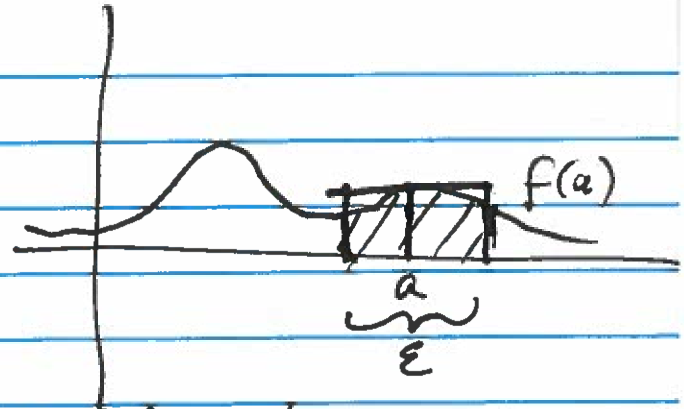
Note: $P(a<X\leq b)=F(b)-F(a)=\int_a^b f(x)\,dx$

Note: $f(x)=\frac{d}{dx}F(x)$.

Densities are not probabilities: they may be greater than 1.



$$P\left(a-\frac{\varepsilon}{2}<X\leq a+\frac{\varepsilon}{2}\right)=\int_{a-\frac{\varepsilon}{2}}^{a+\frac{\varepsilon}{2}} f(x)\,dx \approx \varepsilon f(a)$$



The probability that X is close to a is proportional to f(a), and this approximation improves as ε gets smaller.
So at $\varepsilon=0$, this says $P(X=a)=0$.

For continuous r.v.'s, usually substitute $\int$ for Σ:

$$E[X]=\int_{-\infty}^{+\infty} x f(x)\,dx$$