Midterm exam:
1 page 8.5"x11" of notes, both sides
calculator for arithmetic

Special discrete distributions.

A. X is uniform on $[a,b]$, denoted $X \sim Unif(a,b)$, where a and b are integers iff X is equally probable to be any integer in $[a,b]$.

$p(x) = \frac{1}{b-a+1}$, for $a \le x \le b$.

$E[X] = \frac{1}{2}(a+b)$

$Var(X) = \frac{1}{12}(b-a)(b-a+2)$

depends on formula for sum of consecutive squares

Ex: If X is the outcome of the roll of a fair 6-sided die, $X \sim Unif(1,6)$.

$E[X] = \frac{1}{2}(1+6) = \frac{7}{2}$

$Var(X) = \frac{1}{12}(6-1)(6-1+2) = \frac{35}{12}$

B. A Bernoulli random variable, denoted $X \sim Ber(p)$, is a random indicator variable with

$p(1) = p$ and $p(0) = 1-p$.

$E[X] \stackrel{!}{=} 1 \cdot p + 0(1-p) = p$, $E[X^2] = 1^2 \cdot p + 0^2(1-p) = p$

$Var(X) = E[X^2] - (E[X])^2 = p - p^2 = p(1-p)$

Ex: Coin flip of a coin with prob. p of heads.

c. A binomial random variable, denoted $X \sim Bin(n,p)$, is the sum of n independent $Ber(p)$ random variables. I.e., the number of successes in n Bernoulli trials.

$$p(i) = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } 0 \le i \le n$$

$E[X] = np$

Let $X_i \sim Ber(p)$ be independent, for $1 \le i \le n$, with $X = X_1 + X_2 + \dots + X_n$.

$Var(X_i) = p(1-p)$.

$$Var(X) = Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i) \quad (X_i \text{ are ind})$$

$$= \sum_{i=1}^{n} p(1-p) = np(1-p)$$

Ex: # of heads in n independent flips of a coin that has prob. p of heads.

Slidepack 6, slides 67-80 on error correcting codes.