Equally Likely Outcomes

$$P(E) = \sum_{a\in E} P(a) \quad \text{(axiom 3)}$$

$$= \sum_{a\in E} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}$$

<u>Ex</u>: Assume ♣J face-up. Assume any 5-card hand is equally likely.

$$P(\text{no trump in initial hand}) = \frac{\binom{15}{5}}{\binom{19}{5}}$$

$$= \frac{15\cdot 14\cdot 13\cdot 12\cdot 11}{19\cdot 18\cdot 17\cdot 16\cdot 15} = \frac{3{,}003}{11{,}628} \approx 0.258$$

<u>Ex</u>: Assume your 5-card hand is dealt ~~fa~~ before the trump is turned up.

$$P(\geq 1 \text{ marriage in your hand}) = \frac{\binom{4}{1}\binom{18}{3} - \binom{4}{2}\binom{16}{1}}{\binom{20}{5}}$$

pick a suit for KQ   pick 3 cards

$$\approx 0.204$$

<u>Ex</u>: Assume 365 birthdays are equally probable. What is the prob. that, of n people, none share the same birthday?

Ω = assignments of a birthday to each of n people.

E = " " " unique bday " " " " .

$$P_n = P(\text{no shared birthday among } n \text{ people})$$

$$= \frac{|E|}{|\Omega|} = \frac{P(365,n)}{365^n} = \frac{365!/(365-n)!}{365^n}$$

$$= \frac{365\cdot 364\cdot 363\cdots(365-n+1)}{365^n}$$

Some values:

$n = 23 \Rightarrow p_{23} < 0.5$

$n = 77 \Rightarrow p_{77} < 1/5000$

$n = 100 \Rightarrow p_{100} < 1/3 \times 10^{15}$

<u>Conditional Probability of E given F,</u> written $P(E|F)$, where $F \neq \emptyset$, is the probability that E occurs, given that F occurred. Sample space $\Rightarrow$ F, event E $\Rightarrow$ $E \cap F$.

With equally likely outcomes,

$$P(E|F) = \boxed{\frac{|E \cap F|}{|F|}}$$

$$= \frac{|E \cap F| / |\Omega|}{|F| / |\Omega|} = \boxed{\frac{P(E \cap F)}{P(F)}}$$

This turns out to be the formula even if outcomes aren't equally likely.

Ex: Roll a fair die. What is $P(5 | \text{odd})$?

$E = \{5\}$, $F = \{1, 3, 5\}$

1. From counting: $P(E|F) = \frac{|E \cap F|}{|F|} = \frac{|E|}{|F|} = \frac{1}{3}$
2. From probabilities: $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3}$