Permutations.

How many sequences are there of $\{1, 2, 3, \ldots, n\}$, where each is used exactly once?

$n(n-1)(n-2)\cdots\cdots 3 \cdot 2 \cdot 1 = n!$ $(0! = 1)$

The number of permutations of n distinct objects. (Order matters.)

Generalizing, how many sequences of length k are there, using each of $\{1, 2, 3, \ldots n\}$ 0 or 1 times, where $k \leq n$.

$$P(n,k) = n(n-1)(n-2)\cdots\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

The number of permutations of n distinct objects taken k at a time.

Ex: How many 4-character passwords are there, where each character is either a lowercase letter or a digit, and no character can be repeated?

$$36 \cdot 35 \cdot 34 \cdot 33 = \frac{36!}{32!} = P(36,4).$$

What if the n objects aren't all distinguishable?

Ex. How many permutations of DAWGY? $5!$

" " " DOGGY? $= 5!/2!$

$OG_1DYG_2 = OG_2DYG_1$

How many permutations of GODOGGY? $\frac{7!}{3!2!1!1!}$

Combinations (Order doesn't matter.)

Ex: Your elf-lord avatar can carry any 3 objects chosen from (1) sword, (2) knife, (3) staff, (4) ring, (5) laptop. How many combinations are there?

$$\binom{5}{3} = \frac{P(5,3)}{P(3,3)} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4 \cdot 3}{3!}$$

"number of combinations of 5 objects taken 3 at a time", "5 choose 3"

"binomial coefficients"

More generally, if $0 \le r \le n$,

$$\binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{r!\,(n-r)!}$$

Ex: How many unordered pairs from n objects?

$$\binom{n}{2} = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)}{2} \in \Theta(n^2)$$