Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work briefly*.

- **Bundles:** This week’s turnin bundles: (A) problems 1–3, (B) problems 4–6, (C) problems 7–8. PRINT your full name in the *upper left corner* of each bundle’s top page, with your last name printed clearly in all CAPITAL LETTERS. Each bundle should be stapled separately. We don’t supply the stapler.

A1. Let $X$ be the sum of 20,000 real numbers, and $Y$ be the same sum, but with each number rounded to the nearest integer before summing. If the roundoff errors are independent and uniformly distributed over (-0.5, +0.5), use the Central Limit Theorem to estimate the probability that $|X - Y| > 50$. (Hint: you will need results on the uniform continuous random variable from Example 3.4, page 8 of the text.)

A2. Suppose $X \sim \text{Bin}(1000, 0.3)$ Give upper bounds, to 2 significant digits, on $P(X > 500)$ based on:

(a) Markov’s inequality.

(b) Chebyshev’s inequality.

(c) The Chernoff bound.

A3. Server $A$ sequentially handles 30 jobs, each of whose service times are i.i.d. (independent, identically distributed) with mean 50 milliseconds and standard deviation 10 milliseconds. Server $B$ has an analogous workload, but its 30 jobs each have mean 52 milliseconds and standard deviation 15 milliseconds.

(a) Estimate the probability that server $A$ finishes in less than 1400 milliseconds.

(b) Estimate the probability that server $B$ finishes in less than 1400 milliseconds.

(c) Suppose that $X_i \sim N(\mu_i, \sigma_i^2)$, for $i = 1, 2$, are independent. What is $E[X_1 - X_2]$? What is $\text{Var}[X_1 - X_2]$?

(d) Estimate the probability that server $B$ finishes before $A$. You may use without proof the fact that, if $X_1$ and $X_2$ are independent normal random variables, then $aX_1 + bX_2$ is also normal, for any constants $a$ and $b$.

(e) If you did part (d) correctly, you will discover that $B$ has a significant chance of finishing earlier than $A$, even though $A$ has the smaller mean completion time. Explain how this is possible.
B4. Many people cancel their reservations at hotels at the last minute. To compensate, most hotels overbook when possible. That is, they make more reservations than they have rooms. A particular hotel has 800 rooms and, on average, 20% of the room reservations get cancelled, independently of each other. Suppose that, for a particular night, the hotel has 975 reservations. Approximate the probability that more parties will show up with reservations than they have rooms.

B5. Suppose that accidents on a particular road occur as a Poisson process at a rate of 4 per month. Use the Central Limit Theorem to approximate the probability that, in 48 months, there will be more than 175 accidents.

B6. Statistician Stephanie wants to estimate the mean height \( h \) (in meters) of a population, based on \( n \) independent samples \( X_1, X_2, \ldots, X_n \) chosen uniformly from the population. She uses the sample mean \( M_n = \frac{X_1 + X_2 + \ldots + X_n}{n} \) as the estimate of \( h \) and a rough guess of 0.25 meters for the standard deviation of each sample \( X_i \).

(a) How great should \( n \) be so that the standard deviation of \( M_n \) is at most 1 centimeter?
(b) How great should \( n \) be so that Chebyshev’s inequality guarantees that the estimate is within 3 centimeters of \( h \), with probability at least 0.98?
(c) Stephanie realizes that all people in the population have heights between 1.6 and 2.0 meters. Knowing her statistics, she knows that, when \( X \) is restricted to take values in the interval \([a, b]\), \( \text{Var}(X) \leq (b - a)^2/4 \). She revises her rough guess of the standard deviation of each sample according to this inequality. Given this new estimate, what are the revised values of \( n \) obtained in parts (a) and (b)?

C7. I win 48% of Schnapsen games played against the devilish program Doktor Schnaps. Use the Normal approximation to estimate the probability that, in a random sample of 100 future games I play against Doktor Schnaps:

(a) I will win fewer than 40.
(b) I will win at most 40.
(c) I will win between 44 and 52 (inclusive).

C8. Suppose \( X \sim \text{Poi}(0.3) \). Give upper bounds, to 2 significant digits, on \( P(X \geq 2) \) based on:

(a) Markov’s inequality.
(b) Chebyshev’s inequality.
(c) The Chernoff bound. This bound for a Poisson random variable is:

\[
P(X \geq k) \leq e^{-\lambda} \left( \frac{\lambda e}{k} \right)^k
\]