Review of Important Distributions

1. Discrete
2. Continuous
Discrete Random Variables
Discrete Uniform Distribution

**Definition:** A random variable that takes any integer value in an interval with equal likelihood

**Example:** Choose an integer uniformly between 0 and 10

**Parameters:** integers $a$, $b$ (lower and upper bound of interval)

**Notation:** $X \sim \text{Unif}(a,b)$

**Properties:**

\[
E[X] = \frac{a + b}{2}
\]

\[
\text{Var}(X) = \frac{(b - a)(b - a + 2)}{12}
\]

**pmf:** $P(X=k) = \frac{1}{b-a+1}$ if $k \in [a, b]$, 0 otherwise
Bernoulli Distribution

**Definition:** value 1 with probability $p$, 0 with probability $1-p$

**Example:** coin toss ($p = \frac{1}{2}$ for fair coin)

**Parameters:** $p$

**Notation:** $X \sim \text{Ber}(p)$

**Properties:**

$E[X] = p$

$\text{Var}(X) = p(1-p)$
Binomial Distribution

**Definition:** sum of $n$ independent Bernoulli trials, each with parameter $p$

**Example:** number of heads in 10 independent coin tosses

**Parameters:** $n$, $p$

**Notation:** $X \sim \text{Bin}(n,p)$

**Properties:**

$E[X] = np$

$\text{Var}(X) = np(1-p)$

$p$mf: $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
Poisson Distribution

**Definition:** number of events that occur in a unit of time, if those events occur independently at an average rate $\lambda$ per unit time

**Example:** # of cars at traffic light in 1 minute, # of deaths in 1 year by horse kick in Prussian cavalry

**Parameters:** $\lambda$

**Notation:** $X \sim \text{Poi}(\lambda)$

**Properties:**

$E[X] = \lambda$

$\text{Var}(X) = \lambda$

pmf: $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
Geometric Distribution

**Definition:** number of independent Bernoulli trials with parameter $p$ until and including first success (so $X$ can take values 1, 2, 3, ...)

**Example:** # of coins flipped until first head

**Parameters:** $p$

**Notation:** $X \sim \text{geo}(p)$

**Properties:**

\[
\begin{align*}
\mathbb{E}[X] &= \frac{1}{p} \\
\text{Var}(X) &= \frac{1-p}{p^2} \\
\text{pmf: } P(X = k) &= (1-p)^{k-1}p
\end{align*}
\]
Hypergeometric Distribution

**Definition:** number of successes in \( n \) draws (without replacement) from \( N \) items that contain \( K \) successes in total

**Example:** An urn has 10 red balls and 10 blue balls. What is the probability of drawing 2 red balls in 4 draws?

**Parameters:** \( n, N, K \)

**Properties:**

\[
E[X] = n \cdot \frac{K}{N}
\]

\[
\text{Var}(X) = n \cdot \frac{K(N - K)(N - n)}{N^2(N - 1)}
\]

**pmf:** \( P(X = k) = \frac{(K)_k (N - K)_{n - k}}{(N)_n} \)

Think about the pmf; we've been doing it for weeks now: ways-to-choose-successes times ways-to-choose-failures divided by ways-to-choose-all.

Also, consider that the binomial dist. is the without-replacement analog of this.
Continuous Random Variables
Continuous Uniform Distribution

**Definition:** A random variable that takes any real value in an interval with equal likelihood

**Example:** Choose a real number (with infinite precision) between 0 and 10

**Parameters:** $a, b$ (lower and upper bound of interval)

**Notation:** $X \sim \text{Uni}(a,b)$

**Properties:**

$\mathbb{E}[X] = \frac{a + b}{2}$

$\text{Var}(X) = \frac{(b - a)^2}{12}$

pdf: $f(x) = \frac{1}{b-a}$ if $x \in [a, b]$, 0 otherwise
Exponential Distribution

Definition: Time until next event in Poisson process

Example: How long until the next soldier is killed by horse kick?

Parameters: $\lambda$, the average number of events per unit time

Notation: $X \sim \text{Exp}(\lambda)$

Properties:

$E[X] = \frac{1}{\lambda}$

$\text{Var}(X) = \frac{1}{\lambda^2}$

pdf: $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, $0$ for $x < 0$
Normal Distribution

Description: Classic bell curve

Example: Quantum harmonic oscillator ground state (exact), Human heights, binomial random variables (approximate)

Parameters: $\mu$, $\sigma^2$

Notation: $X \sim N(\mu, \sigma^2)$

Properties:

$E[X] = \mu$

$\text{Var}(X) = \sigma^2$

pdf: $f(x) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$