CSE 312: Foundations of Computing II
Quiz Section #1: Permutations, Combinations, and the Pigeonhole Principle

Review/Mini-Lecture/Main Theorems and Concepts From Lecture

**Product Rule:** Suppose there are $m_1$ possible outcomes for event $A_1$, then $m_2$ possible outcomes for event $A_2$, …, $m_n$ possible outcomes for event $A_n$. Then the total possible outcomes overall is:

**Number of ways to order $n$ distinct objects:**

**Permutations** (number of ways to “pick” $k$ objects out of $n$ when order matters):

**Combinations** (number of ways to “choose” $k$ objects out of $n$ when order doesn’t matter):

**Binomial Theorem:**

**Principle of Inclusion-Exclusion (PIE):**

**Pigeonhole Principle:** If there are $n$ pigeons with $k$ holes and $n > k$, then at least one hole contains at least $p$ pigeons.

**Complementary Counting** (Complementing): If asked to find the number of ways to do X, you can

**Multinomial Coefficients:** Suppose there are $n$ objects, but only $k$ are distinct, with $k \leq n$. (For example, “godoggy” has $n = 7$ objects [characters] but $k = 4$ distinct objects, $\{g, o, d, y\}$). Let $n_i$ for $i = 1, ..., k$ be the number of times object $i$ appears. (For example, $\{3, 2, 1\}$ continuing the “godoggy” example). The number of ways to arrange the $n$ objects is
Exercises

1. In the game of bridge, a hand consists of 13 cards. Given a bridge hand consisting of 5 spades, 2 hearts, 3 diamonds, and 3 clubs, in how many ways can the hand be arranged so that the cards of each suit are together…
   a) … but not necessarily sorted by rank within each suit?
   b) … and each suit is sorted in ascending rank order?
   c) … and each suit is sorted in ascending rank order and the suits are arranged so that the suit colours alternate?

2. How many ways are there to select 5 cards from a standard deck of 52 cards, where the 5 cards contain cards from at most two suits if:
   a. order does not matter
   b. order matters

3. At a dinner party, all of the people present are to be seated at a circular table. Suppose there is a nametag at each place at the table and that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

4. Consider a set of 25 people that form a social network. (The structure of the social network is determined by which pairs of people in the group are “friends”.) How many possibilities are there for the structure of this social network?
5. You want to choose a team of $m$ people for your startup company from a pool of $n$ applicants, and from these $m$ people you want to choose $k$ to be the team managers. You took CSE 312, so you know you can do this in $\binom{n}{m} \binom{m}{k}$ ways. But your CFO comes up with the formula $\binom{n}{k} \binom{n-k}{m-k}$. Before dumping on your CFO, you decide to check his answer against yours.

Give a combinatorial proof that your CFO’s formula agrees with yours.

6. Suppose we have 3 diamonds and 3 hearts from a standard deck. How many ways are there to arrange the cards if they have to alternate suit?

7. There are 80 restaurants on the Ave. 36 serve wine, 30 serve pizza, 32 serve hamburgers, 13 serve wine and pizza, 19 serve pizza and hamburgers, 8 serve wine and hamburgers, and 6 serve all three.

   a) How many restaurants serve none of the three?
   b) How many restaurants can minors dine at (no wine)?
   c) How many restaurants only serve pizza?

8. How many ways are there to have three initials that have two being the same or all three being the same?

9. There are 40 seats and 40 students in a classroom. There are 10 seats in the front row, and 6 students are near-sighted so they must sit in the front row. How many seating arrangements are possible?

10. A chef is preparing desserts for each day of the week (Su, M, ..., F, S). Each day, he prepares one of: apple pie, cake, ice cream sundae, or fruit platter. Since it is someone’s birthday on Thursday, he must prepare cake. However, he cannot serve the same dessert on any two consecutive days. How many dessert combinations are possible?
11. Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These rabbits are to be distributed to 4 different pet stores such that no store gets both a parent and a child. Not every store must receive a rabbit. How many ways can this be done?

12. There are 6 women and 7 men in a ballroom dancing class. If 4 men and 4 women are chosen and paired off, how many pairings are possible?

13. How many ways are there to seat all of the TA’s (9 of us) and Professor Karlin in a row if Professor Karlin cannot sit on either end?

14. For each part, find the minimum number of cards that have to be dealt from a standard 52-card deck to guarantee a:
   a) Single Pair
   b) Two Distinct Pairs
   c) Full House [3 of a kind, and 2 of a kind]
   d) Straight [5 in a row: lowest is A2345, highest is 10JQKA]
   e) Flush [all 5 cards same suit]
   f) Straight Flush [requirements of Straight and Flush]

15. For each part, find the number of ways to get the following when dealt exactly 5 cards from a standard 52-card deck.
   a) Single Pair (5 cards AABCD)
   b) Two Distinct Pairs (5 cards AABBC)
   c) Full House [3 of a kind, and 2 of a kind] (5 cards AAABB)
   d) Straight [5 in a row: lowest is A2345, highest is 10JQKA]
   e) Flush [all 5 cards same suit]
   f) Straight Flush [requirements of Straight and Flush]

16. How many ways are there to arrange 8 people in a circle?