Problem

Toss a red die and a green die. What is the probability that the sum mod 6 is 4 given that the green die shows a 5?

\begin{align*}
\Pr((R + G) \mod 6 = 4 | G = 5) &= \frac{\Pr(G = 5 \text{ and } (R + G) \mod 6 = 4)}{\Pr(G = 5)} = \frac{1}{6} \\
\Pr(G = 5 \text{ and } R = -1 \mod 6) &= \Pr(G = 5 \text{ and } R = 5) = \frac{1}{36}
\end{align*}

Problem

In a group of N people 15% are left-handed. Suppose that 100 times you pick a random person (each person is picked each time with probability 1/N) and ask that person if they are left-handed or not. What is the probability that among the 100 queries, 55 people are left-handed?

\[ \binom{100}{55} (0.15)^{55} (0.85)^{45} \]

Problem

A lie detector is known to be 80% reliable when the person is guilty and 95% reliable when the person is innocent. If a suspect is chosen from a group of suspects of which only 2% have ever committed a crime, and the test indicates that the person is guilty, what is the probability that he is innocent?

\begin{align*}
I &: \text{event that he is innocent} \\
G &: \text{event that the test indicates guilty}
\end{align*}

\[
\Pr(I|G) = \frac{\Pr(G|I)\Pr(I)}{\Pr(G)} = \frac{\Pr(G|I)\Pr(I)}{\Pr(G|I)\Pr(I) + \Pr(G|I^c)\Pr(I^c)} \\
&= \frac{0.05 \cdot 0.98}{0.05 \cdot 0.98 + 0.8 \cdot 0.02}
\]

Problem

There is a population of N people. The number of good guys among these people is \(i\) with probability \(p_i\). Take a sample of \(n\) people from the population. What is the probability that there are \(j\) good guys in the population conditioned on the fact that there are \(k\) good guys in the sample?

\begin{align*}
E_i &= \text{event that there are } i \text{ good guys among } N \\
S_i &= \text{event that there are } i \text{ good guys in sample}
\end{align*}

\[
\Pr(E_j|S_k) = \frac{\Pr(S_k|E_j)\Pr(E_j)}{\Pr(S_k)} = \frac{\binom{N-j}{k} \binom{N-j}{k}}{\binom{N}{k}} \\
\Pr(S_k) &= \sum_j \Pr(S_k|E_j)\Pr(E_j) = \sum_j \left( \binom{N-j}{k} \binom{N-j}{k} \right) p_j
\]