Midterm Review
general

coverage

everything in text chapters 1-2, slides & homework pre-exam (except “continuous random variables,” started last week) is included, except as noted below.

mechanics

closed book; 1 page of notes (8.5 x 11, ≤ 2 sides, handwritten)

I’m more interested in setup and method than in numerical answers, so concentrate on giving a clear approach, perhaps including a terse English outline of your reasoning.

Corollary: calculators are probably irrelevant, but I will send email to the class list tomorrow with final word on whether they are (dis-)allowed
chapter 1: combinatorial analysis

counting principle (product rule)
permutations
combinations
indistinguishable objects
binomial coefficients
binomial theorem
partitions & multinomial coefficients
inclusion/exclusion

pigeon hole principle
sample spaces & events
axioms
complements, Venn diagrams, deMorgan, mutually exclusive events, etc.
equally likely outcomes
chapter 1: conditional probability and independence

conditional probability
chain rule, aka multiplication rule
total probability theorem
Bayes rule     yes, learn the formula
odds (and prior/posterior odds form of Bayes rule)
independence
conditional independence
gambler’s ruin
ch. 2: random variables

discrete random variables
probability mass function (pmf)
expectation of $X$
expectation of $g(X)$ (i.e., a function of an r.v.)
linearity: expectation of $X+Y$ and $aX+b$
variance
cumulative distribution function (cdf)
  \[ \text{cdf as sum of pmf from } -\infty \]

independence; joint and marginal distributions

important examples:
  \[ \text{uniform, bernoulli, binomial, poisson, geometric} \]

\[ \text{know pmf, mean, variance of these} \]
### Some Important (Discrete) Distributions

<table>
<thead>
<tr>
<th>Name</th>
<th>PMF $f(k)$</th>
<th>$E[k]$</th>
<th>$E[k^2]$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform($a, b$)</td>
<td>$f(k) = \frac{1}{(b-a+1)}, k = a, a+1, \ldots, b$</td>
<td>$\frac{a+b}{2}$</td>
<td>$(b-a+1)^2 - 1$</td>
<td>$12$</td>
</tr>
<tr>
<td>Bernoulli($p$)</td>
<td>$f(k) = \begin{cases} 1 - p &amp; \text{if } k = 0 \ p &amp; \text{if } k = 1 \end{cases}$</td>
<td>$p$</td>
<td>$p$</td>
<td>$p(1 - p)$</td>
</tr>
<tr>
<td>Binomial($p, n$)</td>
<td>$f(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, \ldots, n$</td>
<td>$np$</td>
<td>$np(1 - p)$</td>
<td></td>
</tr>
<tr>
<td>Poisson($\lambda$)</td>
<td>$f(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, \ldots$</td>
<td>$\lambda$</td>
<td>$\lambda(\lambda + 1)$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Geometric($p$)</td>
<td>$f(k) = p(1-p)^{k-1}, k = 1, 2, \ldots$</td>
<td>$\frac{1}{p}$</td>
<td>$\frac{2-p}{p^2}$</td>
<td>$\frac{1-p}{p^2}$</td>
</tr>
<tr>
<td>Hypergeometric($n, N, m$)</td>
<td>$f(k) = \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}, k = 0, 1, \ldots, N$</td>
<td>$\frac{nm}{N}$</td>
<td>$\frac{nm}{N} \left( \frac{(n-1)(m-1)}{N-1} + 1 - \frac{nm}{N} \right)$</td>
<td></td>
</tr>
</tbody>
</table>

See also the summary in B&T following pg 528
Calculus is a prereq, but I’d suggest the most important parts to brush up on are:

taylor’s series for $e^x$

sum of geometric series: $\sum_{i \geq 0} x^i = 1/(1-x)$ ($0 \leq x < 1$)
  Tip: multiply both sides by $(1-x)$

$\sum_{i \geq 1} ix^{i-1} = 1/(1-x)^2$
  Tip1: slide # ~13 in “random variables” lecture notes, or text
  Tip2: if it were $\sum_{i \geq 1} ix^{i+1}$, say, you could convert to the above form by dividing by $x^2$ etc.; 1st few terms may be exceptions

integrals & derivatives of polynomials, $e^x$;
chain rule for derivatives; integration by parts
Good Luck!