More counting + pigeonhole principle

BT Section 1.6, Rosen, Section 7.5 Inclusion-exclusion

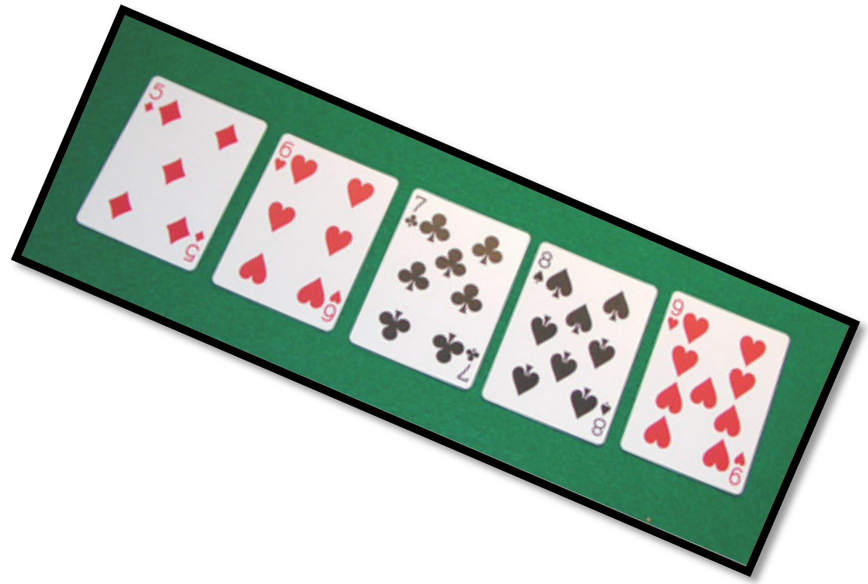
Permutations: Number of ways to order n distinct objects.

$$n! = n \cdot (n-1) \cdots 2 \cdot 1$$

Combinations: Number of ways to choose r things from n things

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

quick review of cards



- 52 total cards
- 13 different **ranks**:
2,3,4,5,6,7,8,9,10,J,Q,K,A
- 4 different **suits**: Hearts, Clubs, Diamonds, Spades

For each object constructed it should be possible to reconstruct the unique sequence of choices that led to it!

Example: How many ways are there to choose a 5 card hand that contains at least 3 aces?

$$\binom{4}{3} \cdot \binom{49}{2}$$

Choose 3 aces, then choose 2 cards from remaining 49.

the sleuth's criterion (Rudich)

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Example: How many ways are there to choose a 5 card hand that contains at least 3 aces?

$$\cancel{\binom{4}{3} \cdot \binom{49}{2}}$$

~~Choose 3 aces, then choose 2 cards from remaining 49.~~

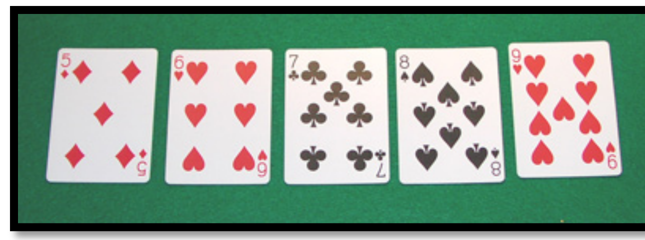
$$\binom{4}{3} \cdot \binom{48}{2} + \binom{4}{4} \cdot \binom{48}{1}$$

counting cards

- How many possible 5 card hands? $\binom{52}{5}$

- A "straight" is five consecutive rank cards of any suit. How many possible straights?

$$10 \cdot 4^5 = 10,240$$



- How many flushes are there?

$$4 \cdot \binom{13}{5} = 5,148$$

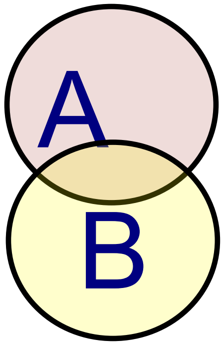
- How many straights that are not flushes?

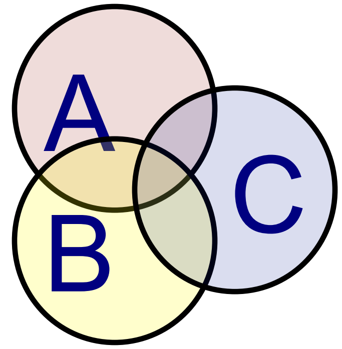
$$10 \cdot 4^5 - 10 \cdot 4 = 10,200$$

- How many flushes that are not straights?

$$4 \cdot \binom{13}{5} - 10 \cdot 4 = 5,108$$

inclusion/exclusion principle





$$|A \cup B| =$$
$$|A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| =$$
$$|A| + |B| + |C|$$
$$-|A \cap B| - |A \cap C| - |B \cap C|$$
$$+ |A \cap B \cap C|$$

General: + singles - pairs + triples - quads + ...

more counting cards

- How many hands have <u>at least</u> three cards of one rank (**three of a kind**)?

$$13 \cdot \binom{4}{3} \cdot \binom{48}{2} + 13 \cdot 48 = 59,280$$

- How many hands are straights or flushes or three of a kind?

Inclusion/exclusion:

Flushes + Straights + 3OfKind

– (Flushes AND Straights) – ~~(Flushes AND 3OfKind)~~ – ~~(Straights AND 3OFKind)~~

\+ ~~(Flushes AND Straights AND 3OfKind)~~

pigeonhole principle

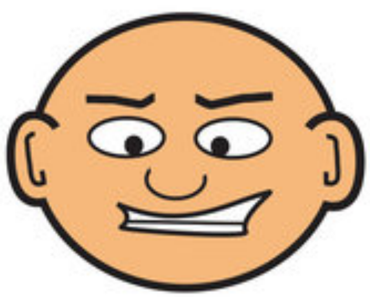
pigeonhole principle

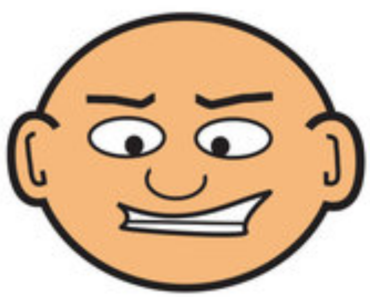
If there are **n** pigeons in **k** holes and **n > k**, then **some hole contains more than one pigeon**.

More precisely, some hole contains at least $\lceil n/k \rceil$ pigeons.

Prove that there are two people in London who have the same number of hairs on their head.

- Londoners have between 0 and 999,999 hairs on their heads.
- There are more than 1,000,000 people in London…





There are many people in this room, some of whom are friends, some of whom are not…

Prove that some two people have the same number of friends.

Pigeons:

Pigeonholes:

Rule for assigning pigeon to pigeonhole: