Inference Rules

Eliminate ∧

$$\frac{A \wedge B}{\therefore \quad A, B}$$

Eliminate ∨

$$\frac{A \vee B, \neg A}{\therefore \quad B}$$

Intro ∧

$$\frac{A; B}{\therefore \quad A \wedge B}$$

Intro ∨

$$\frac{A}{\therefore A \vee B, B \vee A}$$

Direct Proof rule

$$\frac{A \Rightarrow B}{A \rightarrow B}$$

Modus Ponens

$$\frac{P \rightarrow Q; P}{\therefore \quad Q}$$

You can still use all the propositional logic equivalences too!

Intro ∃

$$\frac{P(c) \text{ for some } c}{\therefore \quad \exists x\, P(x)}$$

Eliminate ∃

$$\frac{\exists x P(x)}{\therefore \quad P(c) \text{ for a fresh } c}$$

Eliminate ∀

$$\frac{\forall x\, P(x)}{\therefore P(a) \text{ for any } a}$$

Intro ∀

$$\frac{P(a); a \text{ is arbitrary}}{\therefore \quad \forall x\, P(x)}$$

Excluded Middle

$$\frac{}{\therefore \quad A \vee \neg A}$$

DeMorgan's (Quantifiers)

$$\neg(\forall x\, A) \equiv \exists x(\neg A)$$
$$\neg(\exists x A) \equiv \forall x(\neg A)$$