CSE 311: Foundations of Computing

Lecture 3: Digital Circuits & Equivalence



Recap from last class

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $q \wedge q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

- Associative
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- Distributive
- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- Absorption
- $q \lor (q \land r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- Negation
- $q \lor \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- Double negation

$$- \neg (\neg q) \equiv q$$

Law of implication

$$- q \to r \equiv \neg q \lor r$$

• Identity

- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $q \land q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg(q \lor r) \equiv \neg q \land \neg r$

- Associative
- $(q \lor r) \lor s \equiv q \lor (r \lor s)$
- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$
- Distributive
- $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
- Absorption
- $q \lor (q \land r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- Negation
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- Double negation
- $\neg (\neg q) \equiv q$
- Law of implication
- $q \to r \equiv \neg q \lor r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Associative Identity - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $q \wedge T \equiv q$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ - $q \vee F \equiv q$ Distributive Domination - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor T \equiv T$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ - $q \wedge F \equiv F$ Absorption • Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ - $q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ - $q \lor r \equiv r \lor q$ Double negation - $q \wedge r \equiv r \wedge q$ - $\neg(\neg q) \equiv q$ - De Morgan Laws Law of implication - $\neg (q \land r) \equiv \neg q \lor \neg r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

- $q \rightarrow r \equiv \neg q \lor r$

Example: Show that
$$\neg p \lor (p \lor p) \equiv T$$

 $\neg p \lor (p \lor p) \equiv ($)
 $\equiv ($)
 $\equiv T$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

Associative Identity - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $q \wedge T \equiv q$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ - $q \vee F \equiv q$ Distributive Domination - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor T \equiv T$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ - $q \wedge F \equiv F$ Absorption • Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ - $q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ - $q \lor r \equiv r \lor q$ Double negation - $q \wedge r \equiv r \wedge q$ - $\neg(\neg q) \equiv q$ - De Morgan Laws Law of implication - $\neg (q \land r) \equiv \neg q \lor \neg r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

- $q \rightarrow r \equiv \neg q \lor r$

Example: Show that
$$\neg p \lor (p \lor p) \equiv T$$

 $\neg p \lor (p \lor p) \equiv (\neg p \lor p)$ Idempotent
 $\equiv ()$
 $\equiv T$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

Associative Identity - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $q \wedge T \equiv q$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ - $q \vee F \equiv q$ Distributive Domination - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor T \equiv T$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ - $q \wedge F \equiv F$ Absorption • Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ - $q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ - $q \lor r \equiv r \lor q$ Double negation - $q \wedge r \equiv r \wedge q$ - $\neg(\neg q) \equiv q$ - De Morgan Laws Law of implication - $\neg (q \land r) \equiv \neg q \lor \neg r$ - $q \rightarrow r \equiv \neg q \lor r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

Example: Show that
$$\neg p \lor (p \lor p) \equiv T$$

 $\neg p \lor (p \lor p) \equiv (\neg p \lor p)$ Idempotent
 $\equiv (p \lor \neg p)$ Commutative
 $\equiv T$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

Associative Identity - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $q \wedge T \equiv q$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ - $q \vee F \equiv q$ Distributive Domination - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor T \equiv T$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ - $q \wedge F \equiv F$ Absorption • Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ - $q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ - $q \lor r \equiv r \lor q$ Double negation - $q \wedge r \equiv r \wedge q$ - $\neg(\neg q) \equiv q$ - De Morgan Laws Law of implication - $\neg (q \land r) \equiv \neg q \lor \neg r$

One can prove equivalence between 2 propositional formulas by applying a sequence of elementary equivalences!

- $q \rightarrow r \equiv \neg q \lor r$

Example: Show that
$$\neg p \lor (p \lor p) \equiv T$$

 $\neg p \lor (p \lor p) \equiv (\neg p \lor p)$ | Idempotent
 $\equiv (p \lor \neg p)$ | Commutative
 $\equiv T$ | Negation

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is $\neg p \lor (p \lor p) \equiv \mathsf{T}$

$$\neg p \lor (p \lor p) \equiv (\neg p \lor p) \text{ Idempotent} \\ \equiv (p \lor \neg p) \text{ Commutative} \\ \equiv \mathbf{T} \text{ Negation}$$

A proof is a logical argument that guarantees the conclusion is true. In this case, the conclusion is $\neg p \lor (p \lor p) \equiv \mathsf{T}$

The syntax there is a little terse. In full, it means:

(1) $\neg p \lor (p \lor p) \equiv \neg p \lor p$ by the Idempotent rule, (2) $\neg p \lor p \equiv p \lor \neg p$ by the Commutative rule, and (3) $p \lor \neg p \equiv T$ by the Negation rule.

Therefore, we conclude $\neg p \lor (p \lor p) \equiv \mathbf{T}$

$$\neg p \lor (p \lor p) \equiv (\neg p \lor p) \text{ Idempotent} \\ \equiv (p \lor \neg p) \text{ Commutative} \\ \equiv \mathbf{T} \text{ Negation} \end{cases}$$

Analyzing the Garfield Sentence with a Truth Table

Why not just use a truth table?

q	r	s	¬ <i>s</i>	$r \lor \neg s$	$r \wedge s$	$(r \wedge s) \rightarrow q$	$((r \land s) \rightarrow q) \land (r \lor \neg s)$
F	F	F	Т	Т	F	т	Т
F	F	Т	F	F	F	т	F
F	Т	F	Т	т	F	т	Т
F	Т	Т	F	Т	т	F	F
т	F	F	Т	т	F	т	Т
Т	F	Т	F	F	F	Т	F
Т	т	F	Т	т	F	Т	Т
Т	Т	Т	F	Т	Т	Т	Т

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Show that (q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r
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Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

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Law of implication

- $q \rightarrow r \equiv \neg q \lor r$

- $\neg (q \land r) \equiv \neg q \lor \neg r$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

 $(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$

 $\equiv \\ \equiv \neg q \lor r$

Show that $(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$



- $\neg (q \land r) \equiv \neg q \lor \neg r$ Law of implication
- $\neg (q \lor r) \equiv \neg q \land \neg r \quad q \to r \equiv \neg q \lor r$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

		$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$	
The last two terms are "vacuous truth" maybe the simplify to $\neg q$		$ = (q \land r) \lor [(\neg q \land r) \lor (\neg q \land \neg r)] $ = $(q \land r) \lor [\neg q \land (r \lor \neg r)] $ = =	Associative Distributive
		≡	
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• Identity - $q \wedge T \equiv q$	• Associative • $(q \lor r) \lor s \equiv q \lor (r \lor s)$ • $(q \lor r) \land s \equiv s \land (r \land s)$	≡	
$ q \lor r = q $ Domination $ q \lor T \equiv T $ $ q \land F \equiv F $ Idempotent $ q \lor q \equiv q $ $ q \land q \equiv q $ Commutative $ q \lor r \equiv r \lor q $ $ q \land r \equiv r \land q $ De Morgan Laws	- $(q \land r) \land s \equiv q \land (r \land s)$ • Distributive - $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ • Absorption - $q \lor (q \land r) \equiv q$ • $q \land (q \lor r) \equiv q$ • Negation - $q \lor \neg q \equiv T$ - $q \land \neg q \equiv F$ • Double negation - $\neg (\neg q) \equiv q$	$\equiv \neg q \lor r$	

- $\neg (q \land r) \equiv \neg q \lor \neg r$ Law of implication
- $\neg (q \lor r) \equiv \neg q \land \neg r \quad q \to r \equiv \neg q \lor r$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

		$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$	
The last two terms are "vacuous truth" maybe the simplify to $\neg q$		$ = (q \land r) \lor [(\neg q \land r) \lor (\neg q \land \neg r)] $ = $(q \land r) \lor [\neg q \land (r \lor \neg r)] $ = $(q \land r) \lor [\neg q \land T] $ =	Associative Distributive Negation
		=	
• Identity - $q \land T \equiv q$ - $q \lor F \equiv q$ • Domination - $q \lor T \equiv T$ - $q \land F \equiv F$ • Idempotent - $q \lor q \equiv q$ - $q \land q \equiv q$ • Commutative - $q \lor r \equiv r \lor q$ - $q \land r \equiv r \land q$ • De Morgan Laws	• Associative • $(q \lor r) \lor s \equiv q \lor (r \lor s)$ • $(q \land r) \land s \equiv q \land (r \land s)$ • Distributive • $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ • $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ • Absorption • $q \lor (q \land r) \equiv q$ • $q \land (q \lor r) \equiv q$ • Negation • $q \lor (q \land r) \equiv T$ • $q \land (q \equiv F)$ • Double negation • $\neg (\neg q) \equiv q$	$\equiv \\ \equiv \neg q \lor r$	

- $\neg (q \land r) \equiv \neg q \lor \neg r$ Law of implication
- $\neg (q \lor r) \equiv \neg q \land \neg r \quad q \to r \equiv \neg q \lor r$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

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 $\equiv \neg q \lor r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

$$\begin{array}{c} (q \wedge r) \lor (\neg q \wedge r) \lor (\neg q \wedge \neg r) \\ \equiv (q \wedge r) \lor [(\neg q \wedge r) \lor (\neg q \wedge \neg r)] & \text{Associative} \\ \equiv (q \wedge r) \lor [\neg q \wedge (r \lor \neg r)] & \text{Distributive} \\ \equiv (q \wedge r) \lor [\neg q \wedge T] & \text{Negation} \\ \equiv (q \wedge r) \lor [\neg q] & \text{Identity} \end{array}$$

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- $-q \wedge T \equiv q$
- $-q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $q \land q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg \neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$
- $(q \land r) \land s \equiv q \land (r \land s)$ • Distributive - $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ • Absorption - $q \lor (q \land r) \equiv q$ - $q \land (q \lor r) \equiv q$ • Negation - $q \lor \neg q \equiv T$ - $q \land \neg q \equiv F$ • Double negation - $\neg (\neg q) \equiv q$ • Law of implication - $q \lor r \equiv \neg q \lor r$

Associative

- $(q \lor r) \lor s \equiv q \lor (r \lor s)$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

 $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$ $\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ Associative The last two terms are $\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$ Distributive "vacuous truth" maybe $\equiv (q \wedge r) \vee [\neg q \wedge T]$ Negation the simplify to $\neg q$ $\equiv (q \wedge r) \vee [\neg q]$ Identity $\equiv [\neg q] \lor (q \land r)$ Commutative \equiv q no longer matters in $q \wedge$ r if $\neg q$ automatically Ξ makes the expression true. = • Identity Associative _ - $q \wedge T \equiv q$ - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ $-q \vee F \equiv q$ Distributive Domination $\equiv \neg a \lor r$ - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ $-a \lor T \equiv T$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ - $q \wedge F \equiv F$ Absorption Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ $-q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \lor r \equiv r \lor q$ - $q \wedge \neg q \equiv F$ - $q \wedge r \equiv r \wedge q$ Double negation - De Morgan Laws - $\neg(\neg q) \equiv q$

- $\neg(q \land r) \equiv \neg q \lor \neg r$ • Law of implication

- $q \rightarrow r \equiv \neg q \lor r$

- $\neg (q \lor r) \equiv \neg q \land \neg r$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

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 $\equiv \neg q \lor r$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $\begin{array}{l} \quad q \lor q \equiv q \\ \quad q \land q \equiv q \end{array}$
- *q* ∧ *q* = *q*Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$
- Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation - $\neg(\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

$$\begin{array}{c} (q \wedge r) \lor (\neg q \wedge r) \lor (\neg q \wedge \neg r) \\ \hline \equiv (q \wedge r) \lor [(\neg q \wedge r) \lor (\neg q \wedge \neg r)] \\ \equiv (q \wedge r) \lor [\neg q \wedge (r \lor \neg r)] \\ \hline \equiv (q \wedge r) \lor [\neg q \wedge T] \\ \hline \equiv (q \wedge r) \lor [\neg q] \\ \hline \equiv (q \wedge r) \lor [\neg q] \\ \hline \equiv [\neg q] \lor (q \wedge r) \\ \hline \equiv (\neg q \lor q) \wedge (\neg q \lor r) \\ \hline \equiv \end{array}$$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$ • Idempotent
- $q \lor q \equiv q$
- $q \lor q \equiv q$ $q \land q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$
- Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive $- q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation - $\neg(\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r)$$

$$\equiv (q \land r) \lor [(\neg q \land r) \lor (\neg q \land \neg r)] \quad \text{Associative}$$

$$\equiv (q \land r) \lor [\neg q \land (r \lor \neg r)] \quad \text{Distributive}$$

$$\equiv (q \land r) \lor [\neg q \land T] \quad \text{Negation}$$

$$\equiv (q \land r) \lor [\neg q] \quad \text{Identity}$$

$$\equiv [\neg q] \lor (q \land r) \quad \text{Commutative}$$

$$\equiv (q \lor \neg q) \land (\neg q \lor r) \quad \text{Distributive}$$

$$\equiv (q \lor \neg q) \land (\neg q \lor r) \quad \text{Commutative}$$

$$\equiv$$

 $\equiv \neg q \lor r$

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

 $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$

 $\equiv (q \wedge r) \vee [\neg q \wedge (r \vee \neg r)]$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \wedge$ r if $\neg q$ automatically makes the expression true.

Associative

Distributive

Absorption

Negation

- $q \vee \neg q \equiv T$

- $q \wedge \neg q \equiv F$

- $\neg(\neg q) \equiv q$

- $q \lor (q \land r) \equiv q$

- $q \wedge (q \vee r) \equiv q$

Double negation

Law of implication

- $q \rightarrow r \equiv \neg q \lor r$

- $(q \lor r) \lor s \equiv q \lor (r \lor s)$

- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$

- Identity
- $q \wedge T \equiv q$
- $q \vee F \equiv q$
- Domination
- $-a \lor T \equiv T$ - $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $-q \wedge q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

 $\equiv (q \wedge r) \vee [\neg q \wedge T]$ $\equiv (q \wedge r) \vee [\neg q]$ $\equiv [\neg q] \lor (q \land r)$ $\equiv (\neg q \lor q) \land (\neg q \lor r)$ $\equiv (q \lor \neg q) \land (\neg q \lor r)$ $\equiv T \land (\neg q \lor r)$ \equiv $\equiv \neg q \lor r$ - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$

 $\equiv (q \wedge r) \vee [(\neg q \wedge r) \vee (\neg q \wedge \neg r)]$ Associative Distributive Negation Identity Commutative Distributive Commutative Negation

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
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- $-q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$
- Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation - $\neg(\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

 $[q \land \neg r)]$ Associative Distributive Negation Identity Commutative Distributive Commutative Negation Commutative

Show that
$$(q \land r) \lor (\neg q \land r) \lor (\neg q \land \neg r) \equiv \neg q \lor r$$

The last two terms are "vacuous truth" maybe the simplify to $\neg q$

q no longer matters in $q \land r$ if $\neg q$ automatically makes the expression true.

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
 q ∨ *q* ≡ *q*
- $q \lor q \equiv q$ $q \land q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $-q\wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$
- Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation - $\neg(\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

 $[q \land \neg r)]$ Associative Distributive Negation Identity Commutative Distributive Commutative Negation Commutative Identity

 $(q \land r) \rightarrow (r \lor q)$

• Associative • $(q \lor r) \lor s \equiv q \lor (r \lor s)$ • $(q \land r) \land s \equiv q \land (r \land s)$ • Distributive • $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ • $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ • Absorption

- $q \lor (q \land r) \equiv q$
- $q \wedge (q \vee r) \equiv q$
- Negation
- $-q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- Double negation
- $\neg(\neg q) \equiv q$
- Law of implication
- $q \rightarrow r \equiv \neg q \lor r$

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $q \wedge q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

 $(q \land r) \rightarrow (r \lor q)$

Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ $- q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$

- Negation
- $q \vee \neg q \equiv T$
- $q \wedge \neg q \equiv F$
- Double negation
- $\neg (\neg q) \equiv q$
- Law of implication
- $q \rightarrow r \equiv \neg q \lor r$

Use a series of equivalences:

$$(q \wedge r) \to (r \vee q) \equiv$$

_ Identity - $q \wedge T \equiv q$ - $q \vee F \equiv q$ Domination - $q \lor T \equiv T$ - $q \wedge F \equiv F$ • Idempotent - $q \lor q \equiv q$ - $q \wedge q \equiv q$ Commutative

- $q \lor r \equiv r \lor q$ = - $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace \rightarrow ; move \neg inside; simplify

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Use a series of equivalences:

 $(q \land r) \rightarrow (r \lor q)$

• Associative • $(q \lor r) \lor s \equiv q \lor (r \lor s)$ • $(q \land r) \land s \equiv q \land (r \land s)$ • Distributive • $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ • $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ • Absorption • $q \lor (q \land r) \equiv q$ • $q \land (q \lor r) \equiv q$ • $Q \land (q \lor r) \equiv q$ • Negation • $q \lor \neg q \equiv T$ • $q \land \neg q \equiv F$ • Double negation • $\neg (\neg q) \equiv q$ • Law of implication

- $q \rightarrow r \equiv \neg q \lor r$

Law of Implication

	$(q \land r) \to (r \lor q)$	$\equiv \neg (q \land r) \lor (r \lor q)$
		≡
		≡
-	$\begin{array}{l} \text{Identity} \\ q \wedge T \equiv q \end{array}$	≡
•	$q \lor F \equiv q$ Domination	=
-	$\begin{array}{l} q \lor T \equiv T \\ q \land F \equiv F \end{array}$	
-	$\begin{array}{l} \text{Idempotent} \\ q \lor q \equiv q \end{array}$	=
•	$q \land q \equiv q$ Commutative	
-	$\begin{array}{l} q \lor r \equiv r \lor q \\ q \land r \equiv r \land q \end{array}$	≡
-	De Morgan Laws $\neg (q \land r) \equiv \neg q \lor \neg r$	≡ T
-	$\neg(q \lor r) \equiv \neg q \land \neg r$	

Use a series of equivalences:

Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive $- q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ $- q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation $\neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

 $(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$ $\equiv (\neg q \lor \neg r) \lor (r \lor q)$

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 $(q \land r) \rightarrow (r \lor q)$

Law of Implication DeMorgan

- Identity - $q \wedge T \equiv q$ = - $q \vee F \equiv q$ Domination _ - $q \lor T \equiv T$ - $q \wedge F \equiv F$ = Idempotent - $q \lor q \equiv q$ = - $q \wedge q \equiv q$ Commutative - $q \lor r \equiv r \lor q$ — - $q \wedge r \equiv r \wedge q$ - De Morgan Laws =Т - $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

Use a series of equivalences:

Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive $- q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ $- q \vee (r \wedge s) \equiv (q \vee r) \wedge (q \vee s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation $\neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

 $(q \wedge r) \rightarrow (r \vee q) \equiv \neg (q \wedge r) \vee (r \vee q)$ $\equiv (\neg q \vee \neg r) \vee (r \vee q)$ $\equiv \neg q \vee (\neg r \vee (r \vee q))$

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 $(q \land r) \rightarrow (r \lor q)$

Law of Implication DeMorgan Associative

- Identity
- $q \wedge T \equiv q$
- $q \lor F \equiv q$
- Domination
- $q \lor T \equiv T$
- $q \wedge F \equiv F$
- Idempotent
- $q \lor q \equiv q$
- $q \wedge q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

Use a series of equivalences:

$(q \lor r) \lor s \equiv q \lor (r \lor s)$ $(q \land r) \land s \equiv q \land (r \land s)$ $\overrightarrow{Othermatrix}$ $q \land (r \lor s) \equiv (q \land r) \lor (q \land s)$ $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ $\overrightarrow{Q} \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ $\overrightarrow{Q} \lor (q \land r) \equiv q$ $q \lor (q \land r) \equiv q$ $q \land (q \lor r) \equiv q$ $\overrightarrow{Q} \land (q \lor r) \equiv q$ $\overrightarrow{Q} \lor (q \land r) \equiv q$ $\overrightarrow{Q} \lor (q \lor r) \equiv q$ $\overrightarrow{Q} \lor (q \land r) \equiv q$

Associative

 $(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$ $\equiv (\neg q \lor \neg r) \lor (r \lor q)$ $\equiv (\neg q \lor \neg r) \lor (r \lor q)$ $\equiv \neg q \lor (\neg r \lor (r \lor q))$ $\equiv \neg q \lor ((\neg r \lor r) \lor q)$ $\equiv \neg q \lor ((\neg r \lor r) \lor q)$ \equiv • Domination • q \lor T \equiv T • q \land F \equiv F

 $(q \land r) \rightarrow (r \lor q)$

Law of Implication DeMorgan Associative Associative

Our strategy: Replace \rightarrow ; move \neg inside; simplify

Idempotent
 q ∨ *q* ≡ *q*

- $q \wedge q \equiv q$

- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

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$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

$$(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor (q \lor (\neg r \lor r))$$
$$\equiv \neg q \lor (q \lor (\neg r \lor r))$$

Law of Implication DeMorgan **Associative** Associative Commutative

Associative

Distributive

 Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation

 $- \neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

- $(q \lor r) \lor s \equiv q \lor (r \lor s)$

- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$

- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$

- Ider
- $-q\Lambda$
- a V
- Dor
- q V
- q∧
- Ider
- $q \lor q \equiv q$
- $q \wedge q \equiv q$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace \rightarrow ; move \neg inside; simplify

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$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

$$(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor (q \lor (\neg r \lor r))$$
$$= (\neg q \lor q) \lor (\neg r \lor r)$$
$$\equiv (\neg q \lor q) \lor (\neg r \lor r)$$

Law of Implication **DeMorgan Associative** Associative Commutative Associative

Associative

Distributive

 Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation

 $- \neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

- $(q \lor r) \lor s \equiv q \lor (r \lor s)$

- $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$

- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$

- q∧
- q V
- Do
- q V
- q∧
- Ide
- q V
- $-q\Lambda$
- Commutative
- $q \lor r \equiv r \lor q$
- $q \wedge r \equiv r \wedge q$
- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

Our strategy: Replace \rightarrow ; move \neg inside; simplify

 \equiv

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Т

$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

$$(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv (\neg q \lor q) \lor (\neg r \lor r)$$
$$\equiv (q \lor q) \lor (\neg r \lor r)$$
$$\equiv (q \lor \neg q) \lor (r \lor \neg r)$$
$$\equiv (q \lor \neg q) \lor (r \lor \neg r)$$
$$\equiv (q \lor \neg q) \lor (r \lor \neg r)$$

 $\equiv T$

- $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation $- \neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$ Law of Implication DeMorgan **Associative**

- Associative
- Commutative
- Associative

Commutative (twice)

- De Morgan Laws
- $\neg (q \land r) \equiv \neg q \lor \neg r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

- Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive
 - $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
 - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$
 - Absorption

$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

 $q \wedge$

- q V

- q V - q∧

- q V

- q∧

- q V

 $-q\Lambda$

- $\neg (q \land r) \equiv \neg q \lor \neg r$ - $\neg (q \lor r) \equiv \neg q \land \neg r$

$$(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor (q \lor (\neg r \lor r))$$
$$= (\neg q \lor q) \lor (\neg r \lor r)$$
$$\equiv (q \lor q) \lor (\neg r \lor r)$$
$$\equiv (q \lor \neg q) \lor (r \lor \neg r)$$
$$\equiv \mathbf{T} \lor \mathbf{T}$$

 Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive $- q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation $- \neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

Law of Implication DeMorgan Associative Associative Commutative Associative Commutative (twice) **Negation** (twice)

$$(q \land r) \rightarrow (r \lor q)$$

Use a series of equivalences:

Do

- De

- $\neg (q \lor r) \equiv \neg q \land \neg r$

$$(q \land r) \rightarrow (r \lor q) \equiv \neg (q \land r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv (\neg q \lor \neg r) \lor (r \lor q)$$
$$\equiv \neg q \lor (\neg r \lor (r \lor q))$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv \neg q \lor ((\neg r \lor r) \lor q)$$
$$\equiv (\neg q \lor q) \lor (\neg r \lor r)$$
$$= (q \lor q = q)$$
$$\equiv (q \lor q) \lor ((\neg r \lor r))$$
$$\equiv (q \lor q) \lor (r \lor \neg r)$$
$$\equiv (q \lor r = r \lor q)$$
$$\equiv \mathbf{T} \lor \mathbf{T}$$
$$\equiv \mathbf{T} \lor \mathbf{T}$$
$$\equiv \mathbf{T} \lor \mathbf{T}$$
$$\equiv \mathbf{T} \lor \mathbf{T}$$

 Associative - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ Distributive $- q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$ - $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$ Absorption - $q \lor (q \land r) \equiv q$ - $q \wedge (q \vee r) \equiv q$ Negation - $q \vee \neg q \equiv T$ - $q \wedge \neg q \equiv F$ Double negation $\neg (\neg q) \equiv q$ Law of implication - $q \rightarrow r \equiv \neg q \lor r$

Law of Implication DeMorgan Associative Associative Commutative Associative Commutative (twice) **Negation** (twice) **Domination**/Identity

Logical Proofs of Equivalence/Tautology

- Not smaller than truth tables when there are only a few propositional variables...
- ...but usually *much shorter* than truth table proofs when there are many propositional variables
- A big advantage will be that we can extend them to a more in-depth understanding of logic for which truth tables don't apply.

Lecture 3 Activity

- You will be assigned to **breakout rooms**. Please: •
- Introduce yourself •
- Choose someone to share screen, showing this PDF •
- Show that $p \rightarrow q \equiv \neg q \rightarrow \neg p$ using a sequence of elementary ۲ equivalences.

Fill out a poll everywhere for Activity Credit! Go to pollev.com/philipmg and login with your UW identity

- Associative Identity - $(q \lor r) \lor s \equiv q \lor (r \lor s)$ - $q \wedge T \equiv q$ - $(q \wedge r) \wedge s \equiv q \wedge (r \wedge s)$ $- q \vee F \equiv q$ Distributive Domination - $q \lor T \equiv T$ - $q \wedge F \equiv F$ Absorption Idempotent - $q \lor (q \land r) \equiv q$ - $q \lor q \equiv q$ - $q \wedge (q \vee r) \equiv q$ - $q \wedge q \equiv q$ Negation Commutative - $q \vee \neg q \equiv T$ - $q \lor r \equiv r \lor q$ - $q \wedge \neg q \equiv F$ Double negation - $q \wedge r \equiv r \wedge q$ $\neg (\neg q) \equiv q$ - De Morgan Laws Law of implication - $\neg (q \land r) \equiv \neg q \lor \neg r$ - $q \rightarrow r \equiv \neg q \lor r$
- $\neg (q \lor r) \equiv \neg q \land \neg r$

- $q \wedge (r \vee s) \equiv (q \wedge r) \vee (q \wedge s)$
- $q \lor (r \land s) \equiv (q \lor r) \land (q \lor s)$

Computing With Logic

- **T** corresponds to **1** or "high" voltage
- **F corresponds to 0** or "low" voltage

Gates

- Take inputs and produce outputs (functions)
- Several kinds of gates
- Correspond to propositional connectives (most of them)

Combinational Logic Circuits



Values get sent along wires connecting gates

Combinational Logic Circuits



Values get sent along wires connecting gates

 $\neg p \land (\neg q \land (r \lor s))$

AND Connective vs. AND Gate

 $q \wedge r$

q	r	q∧r
т	Т	Т
Т	F	F
F	Т	F
F	F	F



q	r	OUT
1	1	1
1	0	0
0	1	0
0	0	0







"arrowhead block looks like V"







You may write gates using blobs instead of shapes!







Combinational Logic Circuits



Wires can send one value to multiple gates!

Combinational Logic Circuits



Wires can send one value to multiple gates!

$$(q \land \neg r) \lor (\neg r \land s)$$

Describe an algorithm for computing if two logical expressions/circuits are equivalent.

What is the run time of the algorithm?

Compute the entire truth table for both of them!

There are **2**ⁿ entries in the column for *n* variables.

Logical Proofs of Equivalence/Tautology

- Not smaller than truth tables when there are only a few propositional variables...
- ...but usually *much shorter* than truth table proofs when there are many propositional variables
- A big advantage will be that we can extend them to a more in-depth understanding of logic for which truth tables don't apply.

