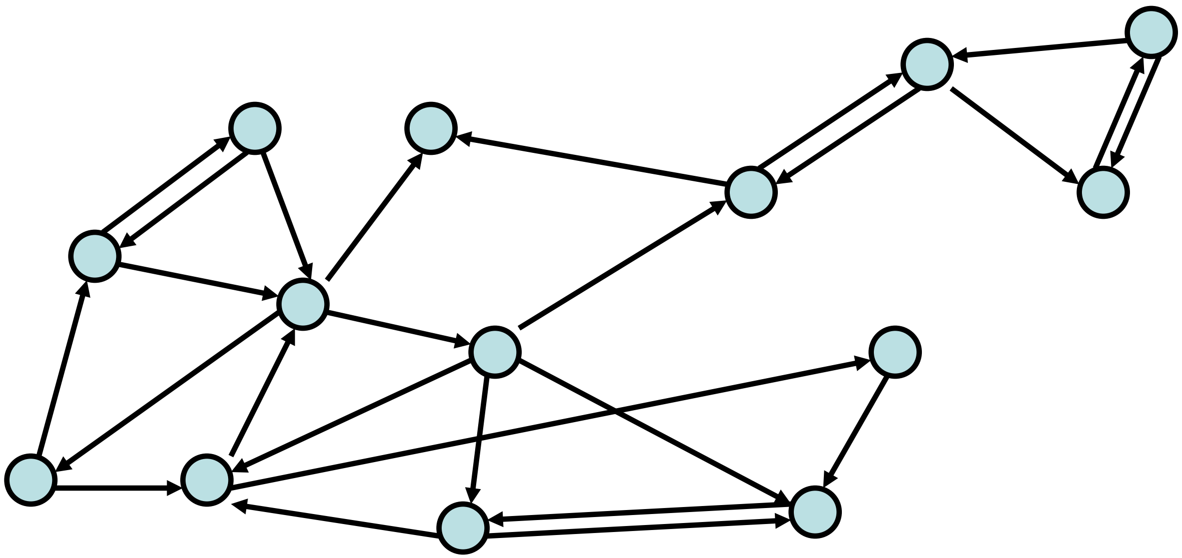
CSE 311: Foundations of Computing

Lecture 27a: Relations and Directed Graphs



Final Exam

Final Exam Practice is up on the website.
We will have two review sessions:

- Thursday from 4:30 – 7:00 in EEB 105
- Sunday from 1:00 – 4:00 in EEB 105

Enjoy!

Epsilon Closure?

One of the major reasons that epsilonClosure was so difficult is that we lacked a way of communicating ideas about the "arrows" in an FSM

Epsilon Closure?

Remember, this course is about the FOUNDATIONS for computing.

We want to give you clean, concise ways of talking about things.

Epsilon Closure?

Last lecture, we talked about functions as a way of discussing infinity.

Now, let's generalize functions.

$$f(x) = y$$

Relations

Let A and B be sets,
A **binary relation from A to B** is a subset of $A \times B$

$R : A \to B$ $\{(1,1), (2,2), (2,3)\}$

Let A be a set,
A **binary relation on A** is a subset of $A \times A$

Relations You Already Know!

$R \subseteq A \times A$

$\geq$ on $\mathbb{N}$

That is: $\{(x,y) : x \geq y \text{ and } x, y \in \mathbb{N}\}$

$<$ on $\mathbb{R}$

That is: $\{(x,y) : x < y \text{ and } x, y \in \mathbb{R}\}$

$=$ on Σ^*

That is: $\{(x,y) : x = y \text{ and } x, y \in \Sigma^*\}$

$\subseteq$ on P(U) for universe U

That is: $\{(A,B) : A \subseteq B \text{ and } A, B \in P(U)\}$

Relation Examples

$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$

$R_2 = \{(x, y) : x \equiv y \pmod 5\}$

$R_3 = \{(c_1, c_2) : c_1 \text{ is a prerequisite of } c_2\}$

(311, 332), (142, 311), (142, 332) ~~(143, 142)~~

$R_4 = \{(s, c) : \text{student } s \text{ had taken course } c\}$

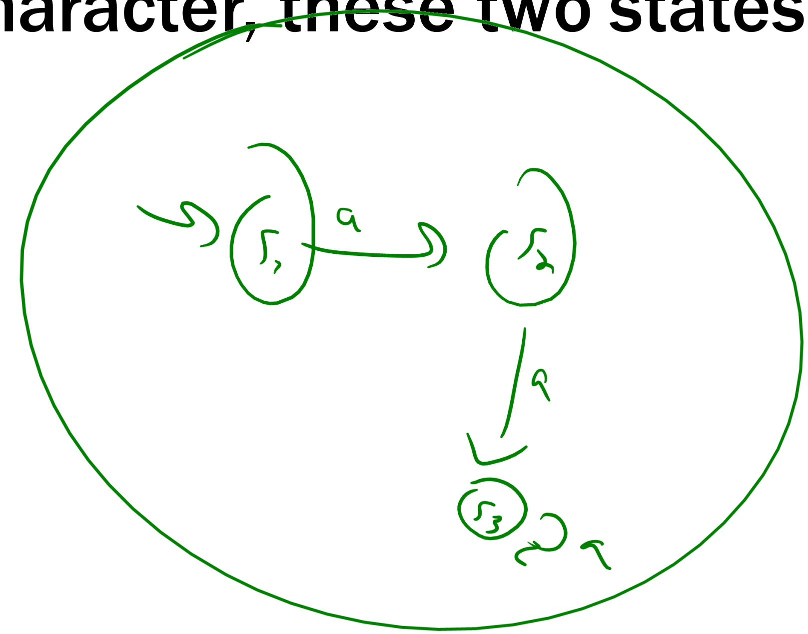
Perhaps most importantly…

The “transitions” in a DFA/NFA are a relation!

They say “for a particular character, these two states are “related”.

$$\{ ((s_1, a), s_2), ((s_2, a), s_3), ((s_3, a), s_3) \}$$

$$R \subseteq (S \times \Sigma) \times S$$



$$S \times \Sigma \times S$$

Properties of Relations

Let R be a relation on A.

R is reflexive iff $(a,a) \in R$ for every $a \in A$

R is symmetric iff $(a,b) \in R$ implies $(b, a) \in R$

R is antisymmetric iff $(a,b) \in R$ and $(b,a) \in R$ implies $a = b$

R is transitive iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Combining Relations

Let R be a relation from A to B.
Let S be a relation from B to C.

$f: A \to B$ $g(f(x))$

$g: B \to C$

The composition of R and S, $S \circ R$ is the relation from A to C defined by:

$$S \circ R = \{(a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

Intuitively, a pair is in the composition if there is a "connection" from the first to the second.

Powers of a Relation

Let R be a relation on A.

$R^2 = R \circ R = \{(a, c) : \exists b\ ((a,b) \in R \text{ and } (b, c) \in R$

$R^0 = \{(a, c) : a \in A\} = A$

$R^1 = \{(a, b) : (a, b) \in R\} = R$

$R^{n+1} = R^n \circ R$

$R = \{(1,2), (2,3), (2,2)\}$

$R^2 = \{(1,3)\}$

$\{(2,2), (1,3), (1,2), (2,3)\}$

$3^0 = 1$

Epsilon Closure…

The epsilonClosure of the epsilon transitions is R^*

We keep on composing the relation over and over until there's nothing left to add.

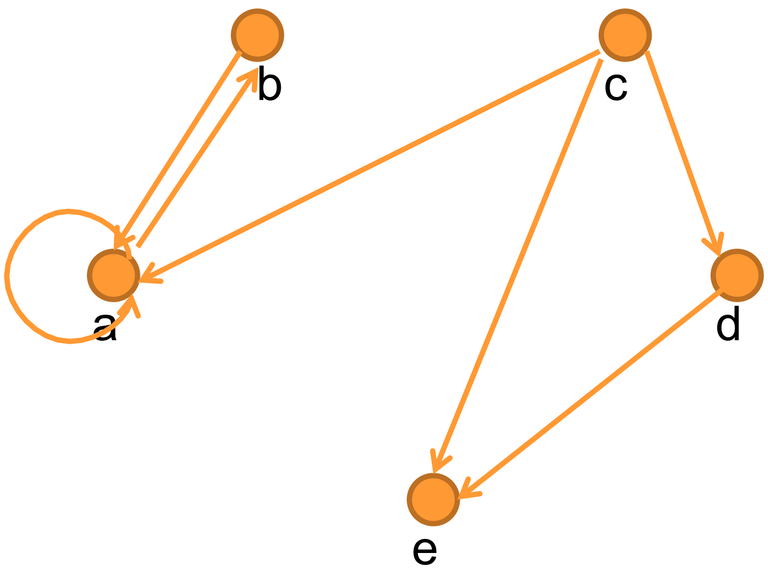
This is called the "transitive closure" of a relation.

$R \cup R^2 \cup R^3 \cup \ldots$

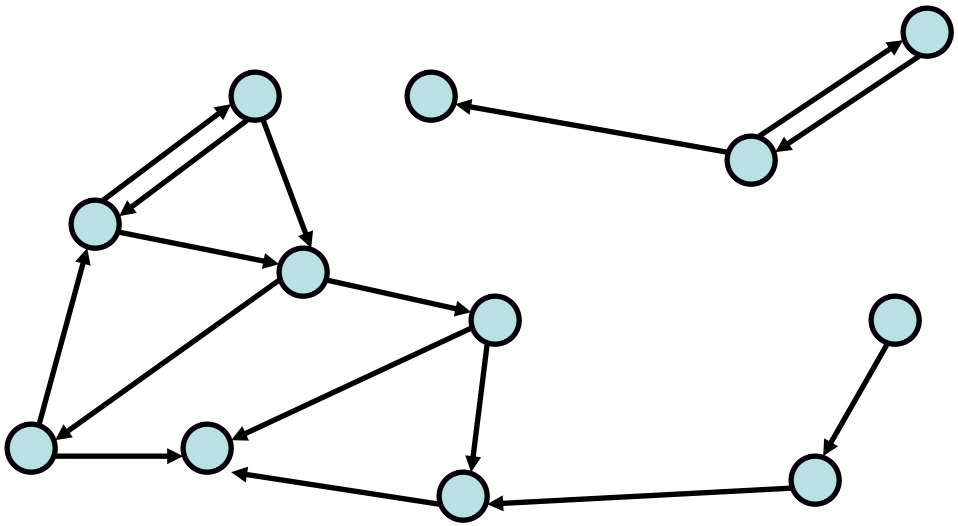
Representation of Relations

Directed Graph Representation (Digraph)

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }



Transitive-Reflexive Closure



Add the minimum possible number of edges to make the relation transitive and reflexive.

The transitive-reflexive closure of a relation R is the connectivity relation R*